Q1.1
1 Point

What is the order of the differential equation
$y^4\cos(y) + 3y\sin(x) + e^{y'} - y^{(3)}\cos(x) = 5$?

(give your answer as a number, for example "2" and not "two")

Highest order of derivative appearing, so 3.

Q1.2
1 Point

True or False? The function $y(x) = xe^x$ is a solution of the differential equation $y' = y + e^x$ on $(-\infty, \infty)$.

- (•) True
- ( ) False

Check that $(xe^x)' = xe^x + e^x$, so $y' = y + e^x$

Q2 Existence and Uniqueness
1 Point

You are given 3 differential equations:

I. $\frac{dy}{dx} = \sqrt{y}$

II. $\frac{dy}{dx} = y\sin(x)$

III. $x\frac{dy}{dx} = y$

For which one(s) of them does the Theorem of Existence and Uniqueness of solutions guarantee that there exists exactly one solution satisfying $y(0) = 0$ defined in an interval containing 0?

For I: $\frac{dy}{dx} = f_1(x,y)$, $f_1(x,y) = \sqrt{y}$. Since $\partial_y f_1 = \frac{1}{2\sqrt{y}}$ is not continuous in any rectangle containing $(0,0)$ in its interior (it is not defined for $y \le 0$), the theorem does not apply.

For II: $\frac{dy}{dx} = f_2(x,y) = y\sin(x)$. Since $f_2(x,y)$, $\partial_y f_2(x,y) = \sin(x)$ are cont. near $(0,0)$ the theorem applies

For III: $x\frac{dy}{dx} = y \Rightarrow \frac{dy}{dx} = \frac{y}{x} = f_3(x,y)$. Since f_3, $\partial_y f_3$ are not defined in any rectangle containing $(0,0)$, the theorem does not apply.

Ans: only II.

Q3

0 Points

You are given the differential equation

$$x\frac{dy}{dx} = (y+1)^2\ln(x) + (y+1)^2 x. \qquad (*)$$

a. Find a general solution of the equation in implicit form.

b. Find a particular solution of the equation defined on an interval containing 1 which satisfies $y(1) = 1$ (you do not need to find the interval).

Your solution can be in implicit form.

c. Check that $y = -1$ is a solution of $(*)$ on $(0, \infty)$. Does $(*)$ have singular solutions?

Show your steps for all parts

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a) $x\frac{dy}{dx} = (y+1)^2(\ln x + x)$

$\Rightarrow \int \frac{dy}{(y+1)^2} = \int\left(\frac{\ln x}{x} + 1\right)dx$ (assuming $y \neq -1$, $x>0$)

$\Rightarrow -\frac{1}{y+1} = \frac{1}{2}(\ln x)^2 + x + C$ $\quad (**)$

b) $y(1) = 1 \Rightarrow$

$-\frac{1}{2} = 1 + C \Rightarrow C = -\frac{3}{2}$

c) Let $y(x) = -1$, then $x\frac{dy}{dx} = 0$. Also $(y+1)^2(\ln x + x) = 0$

So $y(x) = -1$ is a sol'n to $(*)$.

By $(**)$, $y+1 = \dfrac{1}{-\left(\frac{1}{2}(\ln x)^2 + x + C\right)}$ so $y=-1$

would mean $0 = \dfrac{1}{-\left(\frac{1}{2}(\ln x)^2 + x + C\right)}$, which can't happen

for any real C. So $y=-1$ can't be obtained from the general solution and therefore it is a singular solution.