Q1.1
3 Points

Find the largest interval on which the IVP

$$\begin{cases} (x+2)\dfrac{dy}{dx} = \sin(x)y + \ln(|x+5|) \\ y(-3) = 0 \end{cases}$$

has a unique solution according to the Theorem of Existence and Uniqueness.

- ○ $(-2, \infty)$
- ○ $(-\infty, -5)$
- ⊙ $(-5, -2)$
- ○ $(-5, \infty)$
- ○ $(-\infty, \infty)$

Q 1.2 Ans. B

Question 2.1 (5 points)

Find an appropriate substitution $v = f(x, y)$ and use it to reduce the following equation to a **linear** differential equation for v. Then **find an integrating factor for the linear equation.**

$$\frac{dy}{dx} = \sin(x)y + 4\cos^2(x)y^{-5}$$

Write down the linear differential equation for v and the integrating factor for it but **do not solve the equation.**

Bernoulli eq'n, set $v = y^{1+5} = y^6 \Rightarrow y = v^{\frac{1}{6}}$

$$\frac{dy}{dx} = \frac{1}{6} v^{-\frac{5}{6}} \frac{dv}{dx}$$

$$\frac{1}{6} v^{-\frac{5}{6}} \frac{dv}{dx} = \sin(x)\, v^{\frac{1}{6}} + 4\cos^2(x)\, v^{-\frac{5}{6}}$$

$$\Rightarrow \frac{dv}{dx} - 6\sin(x)\, v = 24\cos^2(x)$$

$$p(x) = e^{-\int 6\sin(x)\,dx} = e^{6\cos(x)}$$

Question 2.2 (7 points)

A tank has a total volume of $80lt$ and it is initially filled with $40lt$ of clean water. At time $t = 0$ min, water contaminated with a pollutant having concentration 0.5% starts flowing in at a rate of $4lt/min$ and the well mixed liquid exits the tank at a rate of $6lt/min$.

a) After how many minutes does the tank become full?

b) Write down a differential equation for the amount in liters $q(t)$ of pollutant in the tank at time $t\ min$. **Do not solve the equation.**

c) Which one(s) of the following characterizations apply to the equation you found?

1. Linear
2. Separable
3. Autonomous
4. Bernoulli
5. Homogeneous

a) volume: $v(t) = 40 + 4t - 6lt = 40 - 2t$

so it never becomes full, volume decreases.

(this wasn't originally meant as a trick question, the outflowing rate was bigger than the inflow rate due to oversight).

b)
$$\frac{dq}{dt} = c_{in}\underbrace{r_{in}}_{=4} - c_{out}\underbrace{r_{out}}_{=6}$$

$$= \frac{0.5}{100} \cdot 4 - 6\,\frac{q(t)}{\underbrace{40-2t}_{\text{volume}}}.$$

c) linear eqn.