Plan: Worksheet.

Thm

well approximated by the linearized system at (x_0, y_0)

Suppose: given an Almost Linear System at (x_0, y_0) ((x_0, y_0) is a CP).
Let λ_1, λ_2 be the e-values of linearized system at (x_0, y_0).

I. If λ_1, λ_2 equal, real then: CP (x_0, y_0) is either a node or spiral

As. stable if $\lambda_1 = \lambda_2 < 0$

Unstable if $\lambda_1 = \lambda_2 > 0$

Intuitive explanation: the phase plane portrait of the non-linear system should behave in the same way near a critical point as the phase plane portrait of a linear system which is a perturbation of the linearization of the linear system

Ex: Non-linear system, c.p. at $(0,0)$.

$$\begin{cases} x' = x + y + x^2 \\ y' = 4x + y - 2xy \end{cases}$$

Linearization at $(0,0)$

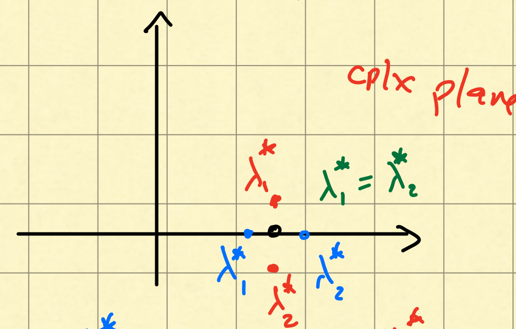
$$\begin{cases} u' = u + v \\ v' = 4u + v \end{cases}$$

Perturbed system which is still linear

$$\begin{cases} u' = 1.001u + 0.9v \\ v' = 4.012u + 1.1v \end{cases}$$

e-values of linearized system

E-values of perturbed linear system can be either of the colored pairs

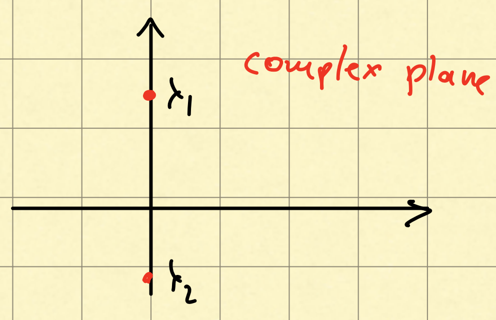


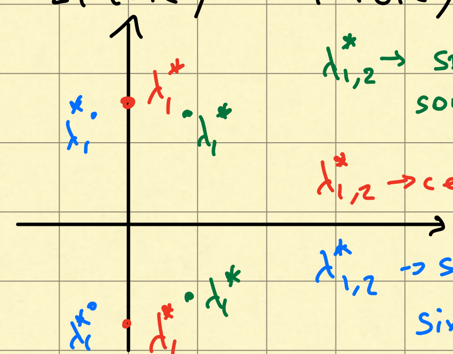
$\lambda_{1,2}^* \to$ node

$\lambda_{1,2}^* \to$ spiral

$\lambda_{1,2}^* \to$ node

2 If λ_1, λ_2 purely imaginary <u>then</u> CP is either center <u>or</u> spiral (can be stable, as. stable, unstable)





$\lambda_{1,2}^* \to$ spiral source

$\lambda_{1,2}^* \to$ center

$\lambda_{1,2}^* \to$ spiral sink

3. In all other cases: CP of same type & stability of linearized system.