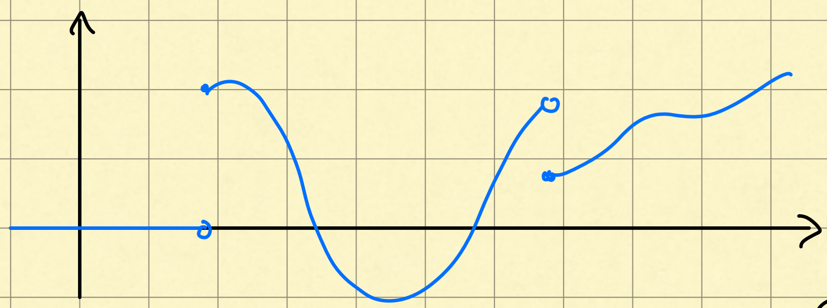
Last time: piecewise cont. fcts (ex: signals w/ time delay, or signals that stop)



Recall: $u_a(t) = u(t-a) = \begin{cases} 1, & t \geq a \\ 0, & t < a \end{cases}$

Ex: $f_3(t) = \begin{cases} \sin(2t), & \pi \leq t < 2\pi \\ 0, & \text{otherwise.} \end{cases}$

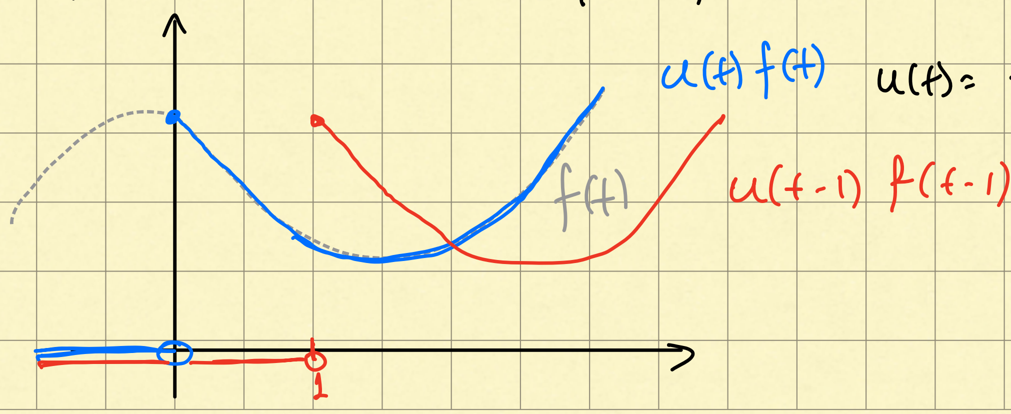
wrote $f_3(t) = \big(u(t-\pi) - u(t-2\pi)\big)\sin(2t)$

Rule: $\mathcal{L}\{u(t-a)\, f(t-a)\} = e^{-as}\, \mathcal{L}\{f(t)\}$

$\Leftrightarrow \mathcal{L}^{-1}\{e^{-as} F(s)\} = u(t-a)\, f(t-a)$

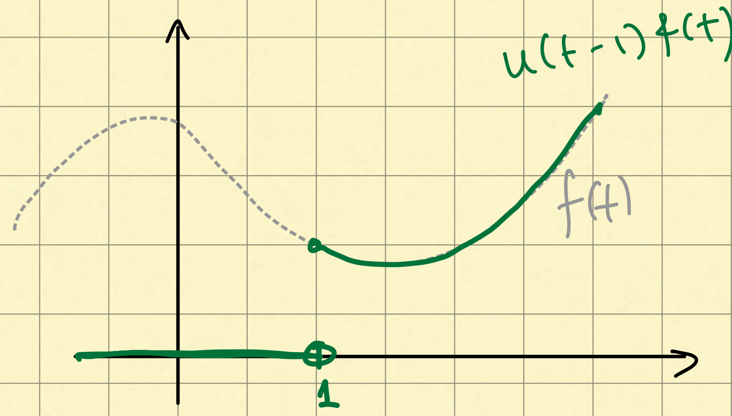
$F(s) = \mathcal{L}\{f(t)\}$

Understand $u(t-a)\, f(t-a)$

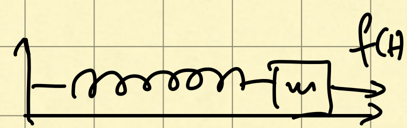


$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

Notice: $u(t-1)\,f(t)$



<u>Application/example.</u>



Spring-mass system, no damping

$m = 1$

$c = 0$

$k = 9$

External force $f(t) = \sin(2t)$ starting at $t = \pi$, stopping at $t = 2\pi$

Eq'n of motion for displacement from equil:

$$m x'' + c x' + k x = f(t)$$

Initially at rest: $x(0) = x'(0) = 0$.

1. Find $X(s) = \mathcal{L}\{x(t)\}$

steps $\begin{cases} \text{a. write } f(t) \text{ w/ step fcts} \\ \text{b. use } \mathcal{L}\{u(t-a)\,f(t-a)\} = e^{-as}\mathcal{L}\{f(t)\} \\ \text{c. } \mathcal{L}\{\sin(at)\} = \dfrac{a}{s^2+a^2} \end{cases}$

Find $\mathcal{L}\{f(t)\} = \mathcal{L}\{(u(t-\pi) - u(t-2\pi))\sin(2t)\}$

$$= \mathcal{L}\{u(t-\pi)\sin(2t)\} - \mathcal{L}\{u(t-2\pi)\sin(2t)\}$$

$$= \mathcal{L}\{u(t-\pi)\sin(2(t-\pi)+2\pi)\}$$

$$-\mathcal{L}\{u(t-2\pi)\sin(2(t-2\pi)+4\pi)\}$$

sin is 2π-periodic

$$= \mathcal{L}\{u(t-\pi)\sin(2(t-\pi))\}$$

$$-\mathcal{L}\{u(t-2\pi)\sin(2(t-2\pi))\}$$

Rule

$$= e^{-\pi s}\mathcal{L}\{\sin(2t)\} - e^{-2\pi s}\mathcal{L}\{\sin(2t)\}$$

$$= e^{-\pi s}\frac{2}{s^2+4} - e^{-2\pi s}\frac{2}{s^2+4}.$$

<u>Therefore:</u> $x'' + 9x = f(t) \Rightarrow$

$$X(s)(s^2+9) = (e^{-\pi s} - e^{-2\pi s})\frac{2}{s^2+4}$$

$$\Rightarrow X(s) = 2\frac{1}{s^2+9}\frac{1}{s^2+4}(e^{-\pi s} - e^{-2\pi s}) //$$