Reminder: - Midterm 2 next Tuesday
- Review worksheet Friday
- No OH today

OH 5.30-8.30 Tuesday

Term-by-term dif'tion of F.S.

If → f cont. for all t
→ $2L$ periodic
→ f' piecewise smooth (f', f'' piecewise smooth)

then F.S. of f can be dif'ted term by term.

i.e.

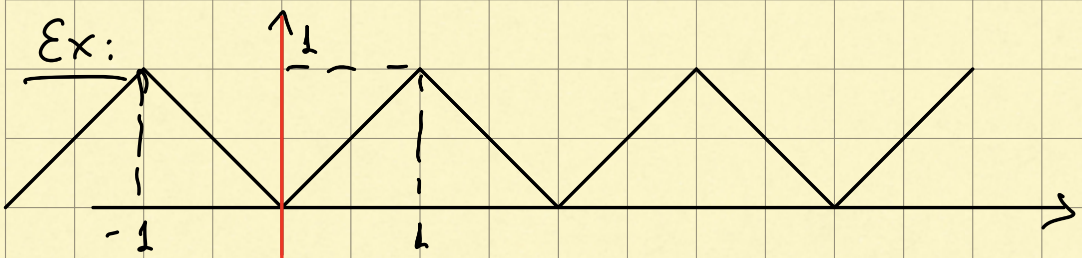
$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{L} t\right) + b_n \sin\left(\frac{n\pi}{L} t\right)$$

then

$$f'(t) = \sum_{n=1}^{\infty} \left(-\frac{n\pi}{L} a_n \sin\left(\frac{n\pi}{L} t\right) + \frac{n\pi}{L} b_n \cos\left(\frac{n\pi}{L} t\right) \right)$$

[at pts where f' not cont. F.series converges to average of side limits].

Ex:



Period 2, even

$L=1$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{\pi n}{1} t\right) \quad \text{(even)}$$

$$a_0 = \frac{1}{1}\int_{-1}^{1} f(t)\,dt = 2\int_0^1 f(t)\,dt = 2\int_0^1 t\,dt = 1.$$

Check:

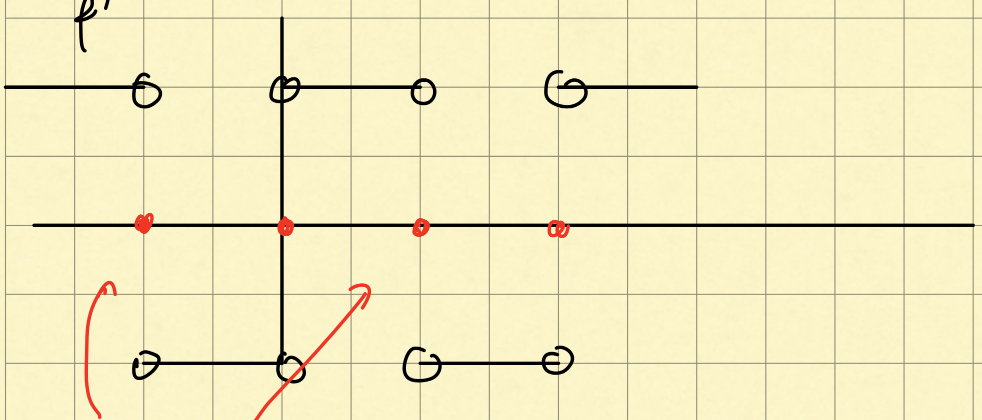
$$a_n = \frac{2}{1}\int_0^1 t\cos(n\pi t)\,dt$$

$$= \dots = 2\,\frac{1+(-1)^n}{\pi^2 n^2}$$

So: $f(t) = \frac{1}{2} + \sum_{n=1}^{\infty} 2\,\frac{1+(-1)^n}{\pi^2 n^2}\cos(\pi n t)$

f : periodic, cont.

f'



1

define it to be average of side limits.

f' piecewise smooth.

Term-by-term dif'tion is valid.

$$f'(t) = \sum_{n=1}^{\infty} (-2) \frac{1+(-1)^n}{\pi n} \sin(\pi n t) \quad (2)$$

Exercise: Take F.S. of (1), see that you find (2).

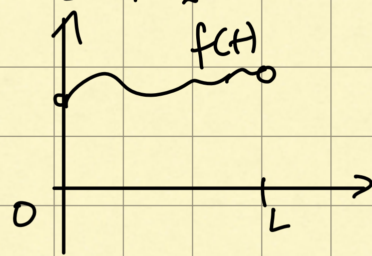
Want: Solve endpoint problems using F.S.

$$\begin{cases} ax'' + bx' + cx = f(t) \\ x(0) = x(L) = 0 \end{cases}$$

or

$$\begin{cases} ax'' + bx' + cx = f(t) \\ x'(0) = x'(L) = 0 \end{cases}$$

$f(t)$ piecewise smooth on $[0, L]$

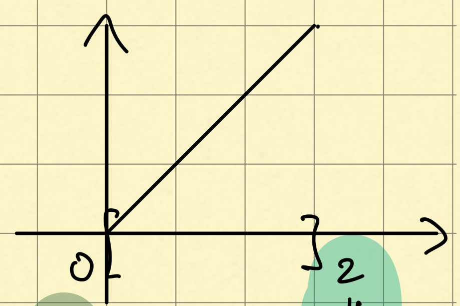


Strategy:

- → Extend f to a periodic fct. Typically take even or odd extension. (Want period $2L$)
- → Take F.S. of extended function.
- → Assume that x has F.S. expansion which can be dif'ted term by term, twice
- → Compute F.S. of LHS of ODE, match terms w/ F.S. of RHS to determine coef. in F.S. of x.
- →. If endpt conditions are satisfied by x then we get a "formal sol'n" x as

a F.S. $f(t)$

Ex: $\begin{cases} x'' + 2x = t \\ x(0) = x(2) = 0 \end{cases}$



Assume

(3) $$x \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{2} t\right) + b_n \sin\left(\frac{\pi n}{2} t\right)$$ $2 = L$

We would be happy if $a_n = 0$ for all n

bec. $\sin\left(\frac{\pi n}{2} 0\right) = \sin\left(\frac{\pi n}{2} \cdot 2\right) = 0.$

So: we will extend f in a way to make this happen.

Try: odd extension for $f(t)$, i.e. Fourier sine series.

$$f(t) = \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{(-1)^{n+1}}{n} \sin\left(\frac{\pi n}{2} t\right)$$ (check!)

Take x'' term by term:

$$x'' \sim \sum_{n=1}^{\infty} \left(-a_n \left(\frac{\pi n}{2}\right)^2 \cos\left(\frac{n\pi}{2} t\right) - b_n \left(\frac{\pi n}{2}\right)^2 \sin\left(\frac{n\pi}{2} t\right)\right)$$

Plug into $x'' + 2x = t$

$$2 \cdot \frac{a_0}{2} + \sum_{n=1}^{\infty} \underbrace{\left(-a_n \left(\frac{\pi n}{2}\right)^2 + 2a_n\right)}_{(*)} \cos\left(\frac{n\pi}{2} t\right)$$

(*)

$$+ \sum_{n=1}^{\infty} \left(-b_n \left(\frac{\pi n}{2}\right)^2 + 2b_n \right) \sin\left(\frac{n\pi}{2} t\right)$$

$$= \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{(-1)^{n+1}}{n} \sin\left(\frac{\pi n}{2} t\right)$$

No cosine terms on RHS:

$$a_n \left(-\left(\frac{\pi n}{2}\right)^2 + 2 \right) = 0 \Rightarrow a_n = 0$$

$$a_0 = 0$$

If take sine series for RHS and ODE $x'' + ax = f$ then x will have sine series.

For b_n:

$$b_n \left(-\left(\frac{\pi n}{2}\right)^2 + 2 \right) = \frac{4}{\pi} \frac{(-1)^{n+1}}{n}$$

$$\Rightarrow b_n = \frac{4}{\pi} \frac{(-1)^{n+1}}{n} \frac{1}{-\left(\frac{\pi n}{2}\right)^2 + 2}$$

So:

$$x(t) = \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{(-1)^{n+1}}{n} \frac{1}{2 - \left(\frac{\pi n}{2}\right)^2} \sin\left(\frac{\pi n}{2} t\right)$$

Note: $x(0) = x(2) = 0$.