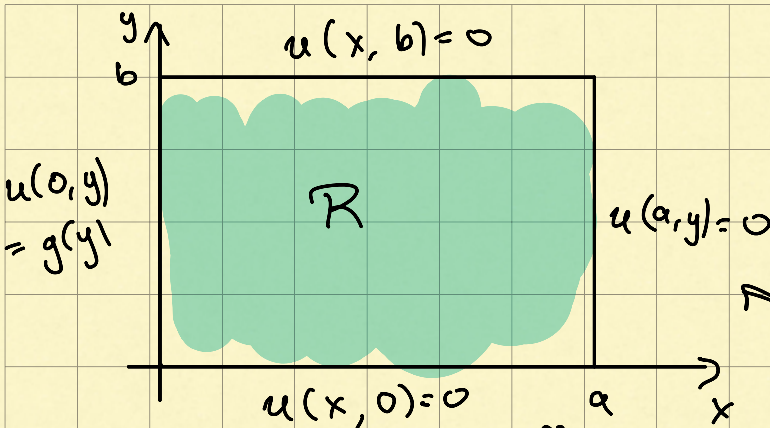

$\nabla^2 u = 0$ in R

$\iff \partial_x^2 u + \partial_y^2 u = 0.$

with boundary cond.

Seek sol'n $u = \sum_{n=0}^{\infty} c_n u_n(x,y)$

where $\nabla^2 u_n = 0$

and $u_n(x,b) = 0$, $u_n(a,y) = 0$, $u_n(x,0) = 0$ $\quad (*)$

(they satisfy the homog. bd conditions),

Assumed: $u_n(x,y) = X_n(x)\, Y_n(y)$ $\quad (\square)$

Found:

$$\frac{X_n''}{X_n} = -\frac{Y_n''}{Y_n} = \lambda. \qquad \lambda \text{ const.}$$

$$\Rightarrow \begin{cases} Y_n'' + \lambda Y_n = 0 \\ X_n'' = \lambda X_n \end{cases}$$

What endpt conditions should X_n, Y_n satisfy?

(Plug $(\square)$ into $(*)$)

$X_n(a) Y_n(y) = 0 \Rightarrow X_n(a) = 0$

$X_n(x) Y_n(b) = 0 \Rightarrow Y_n(b) = 0$

$X_n(x)\,Y_n(0) = 0 \Rightarrow Y_n(0) = 0$

So: $\begin{cases} Y_n'' + \lambda Y_n = 0 \\ Y_n(0) = Y_n(b) = 0. \end{cases}$

start w/ problem for which we have 2 endpt conditions

For what λ is there a non-trivial sol'n?

Seen: Non-trival sol'n exists exactly when $\lambda_n = \left(\frac{n\pi}{b}\right)^2$, $n = 1, 2, \ldots$

Sol'n: $Y_n(y) = \sin\left(\frac{n\pi}{b} y\right)$

see notes from heat eq'n

Now: $\begin{cases} X_n'' = \lambda_n X_n \Rightarrow X_n'' = \left(\frac{n\pi}{b}\right)^2 X_n \\ X_n(a) = 0 \end{cases}$

So: general sol'n $X_n(x) = A\, e^{\frac{n\pi}{b}x} + B\, e^{-\frac{n\pi}{b}x}$

(can also write $X_n(x) = \tilde{A} \cosh\left(\frac{n\pi}{b} x\right) + \tilde{B} \sinh\left(\frac{n\pi}{b} x\right)$)

$$X_n(a) = 0 \Rightarrow A e^{\frac{n\pi}{b}a} + B e^{-\frac{n\pi}{b}a} = 0$$

$$\Rightarrow A = -B e^{-2\frac{n\pi}{b}a}$$

$$X_n(x) = B\left(-e^{\frac{n\pi}{b}x} e^{-2\frac{n\pi}{b}a} + e^{-\frac{n\pi}{b}x}\right)$$

$$= \underbrace{-B e^{-\frac{n\pi a}{b}}}_{\text{doesn't depend on } x.} \left(e^{\frac{n\pi(x-a)}{b}} - e^{-\frac{n\pi}{b}(x-a)}\right)$$

<u>So:</u> can take

$$u_n(x,y) = \underbrace{\left(e^{\frac{n\pi(x-a)}{b}} - e^{-\frac{n\pi}{b}(x-a)}\right)}_{-\frac{1}{Be^{-\frac{n\pi a}{b}}} X_n(x)} \underbrace{\sin\left(\frac{n\pi}{b}y\right)}_{Y_n(y)}$$

$$= 2\sinh\left(\frac{n\pi}{b}(x-a)\right)\sin\left(\frac{n\pi}{b}y\right)$$

2 is absorbed into c_n.

and

$$u(x,y) = \sum_{n=1}^{\infty} c_n \sinh\left(\frac{n\pi}{b}(x-a)\right)\sin\left(\frac{n\pi}{b}y\right)$$

<u>Now:</u> $u(0,y) = g(y)$ (non-hom. bd condition)

$$\sum_{n=1}^{\infty} \left[c_n \sinh\left(\frac{n\pi}{b}(-a)\right) \right] \sin\left(\frac{n\pi}{b} y\right)$$

$$= g(y)$$

<u>Want:</u>

$$c_n \sinh\left(\frac{n\pi}{b}(-a)\right) = \frac{2}{b} \int_0^b g(y) \sin\left(\frac{n\pi}{b} y\right) dy$$

$$\Rightarrow c_n = \frac{2}{b \sinh\left(\frac{n\pi}{b}(-a)\right)} \int_0^b g(y) \sin\left(\frac{n\pi}{b} y\right) dy$$