Plan:

- Investigate the geometric behavior of trajectories of solutions to systems w/ two real eigenvalues, distinct. (from 5.3, haven't finished 5.2 yet)

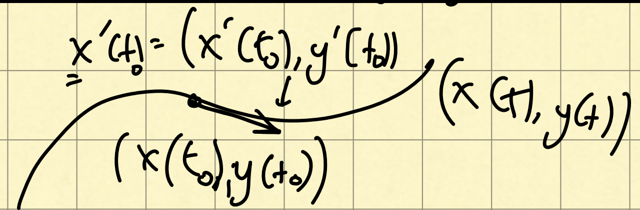
Announcements: OH posted
HW due Tuesday on MyLab

Saw: systems $\underline{\underline{x}}' = \underline{\underline{A}}\,\underline{\underline{x}}$, $\underline{\underline{A}}$ real, distinct e-values

Focus on $\underline{\underline{A}}$ 2 x 2 const. matrix.

write: $\underline{\underline{x}}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$. Can think of $(x(t), y(t))$ as a curve on the x-y plane.

What do those curves look like?



$\underline{\underline{x}}'$ given by dif. eq'n.

Plot velocity vectors of sol'n curves using $\underline{\underline{x}}' = \underline{\underline{A}}\,\underline{\underline{x}}$, relate picture w/ eigenvalues of $\underline{\underline{A}}$. Such a plot is called a phase plane portrait.

<u>Today:</u> real distinct e-values.

<u>Ex:</u> E-values of opposite sign.

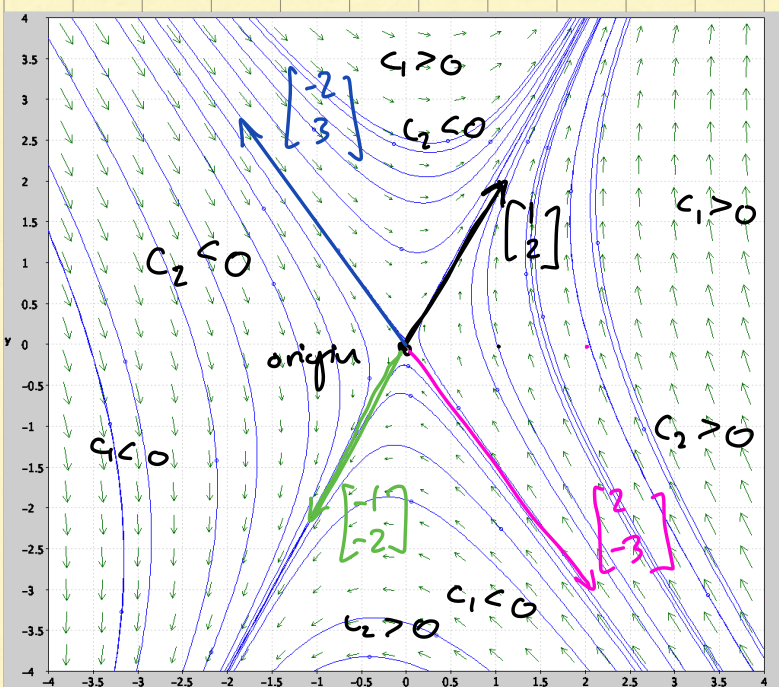
$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \underline{A} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \quad \underline{A} = \begin{bmatrix} -\frac{5}{7} & \frac{6}{7} \\ \frac{18}{7} & -\frac{2}{7} \end{bmatrix}$$

e-values: $1, -2$.

Gen. sol: $\underline{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1 e^{t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{-2t} \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

(check)

<u>Velocity:</u> $\underline{x}'(t) = c_1 e^{t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} - 2 c_2 e^{-2t} \begin{bmatrix} 2 \\ -3 \end{bmatrix}$



As $t \to \infty$, approx. get large multiple of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, positive or negative, if $c_1 \neq 0$.

If $c_1 = 0$
$\underline{x}'(t) = -2c_2 e^{-2t} \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

As $t \to -\infty$, multiple of $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ approx.

In case of real e-values of opposite signs the origin is a <u>saddle point</u>

Ex 2 dinstinct, negative e-values.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underline{\underline{A}} \begin{bmatrix} x \\ y \end{bmatrix}, \quad \underline{\underline{A}} = \begin{bmatrix} -\frac{25}{7} & \frac{2}{7} \\ \frac{6}{7} & -\frac{27}{7} \end{bmatrix}$$

e-values: $-3, -4$

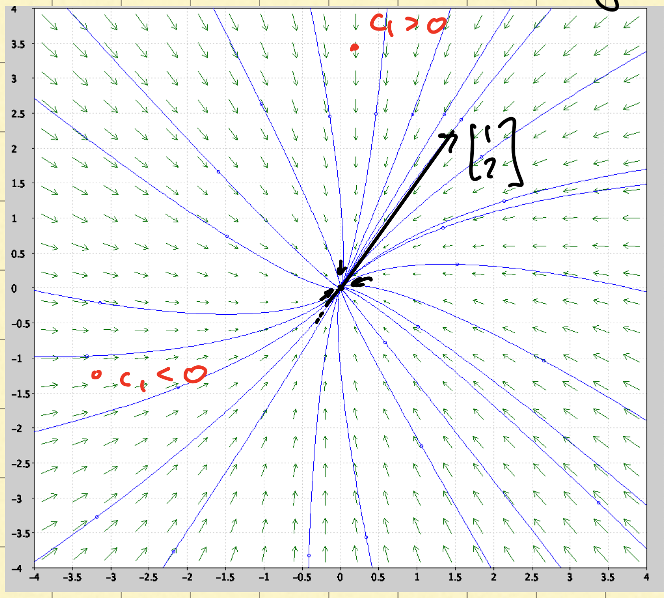
Sol: (check)

$$\underline{\underline{x}}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1 e^{-3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} 2 \\ -3 \end{bmatrix} \quad (1)$$

$$\underline{\underline{x}}'(t) = -3c_1 e^{-3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} - 4c_2 e^{-4t} \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$= e^{-3t} \left(-3c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \underbrace{4c_2 e^{-t} \begin{bmatrix} 2 \\ -3 \end{bmatrix}}_{\to 0 \text{ as } t \to \infty} \right)$$

As $t \to \infty$ approaching multiple of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$



All: $\underline{\underline{x}}(t) \to 0$ as $t \to \infty$

(nodal sink)

Ex Distinct Positive e-values.

$$\underline{\underline{x}}' = \underline{\underline{\tilde{A}}}\,\underline{\underline{x}} \quad (2) \qquad \underline{\underline{\tilde{A}}} = \begin{bmatrix} \frac{25}{7} & -\frac{2}{7} \\ -\frac{6}{7} & \frac{27}{7} \end{bmatrix} \quad \text{e.v. } 3, 4.$$

(compare w/ previous ex.)

Time reversal.

const. matrix.

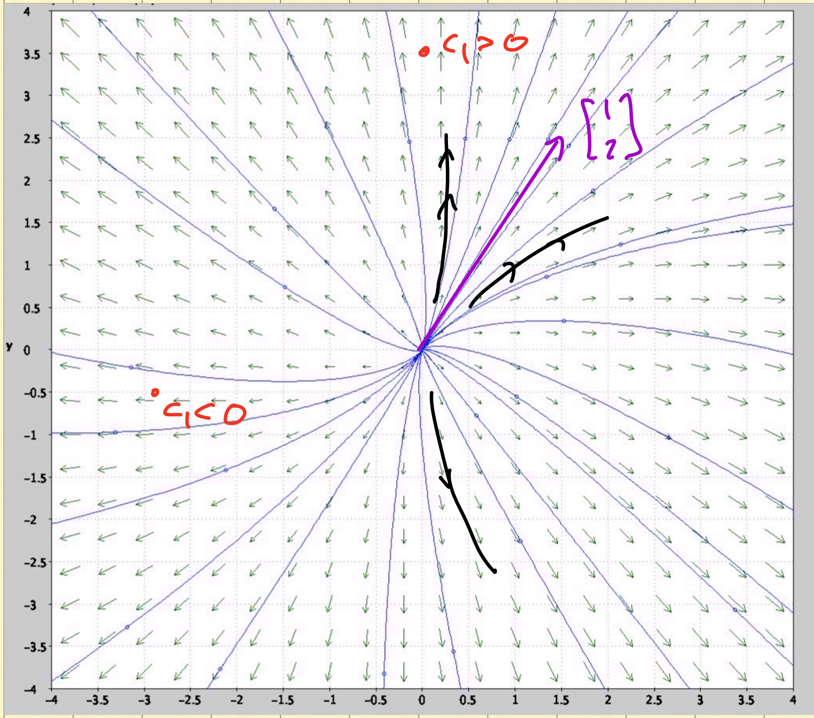
Given sol'n to $\underline{\underline{x}}' = \underline{\underline{A}}\,\underline{\underline{x}}$, set $\underline{\underline{\tilde{x}}}(t) = \underline{\underline{x}}(-t)$

$$\underline{\underline{\tilde{x}}}'(t) = -\underline{\underline{x}}'(-t) = -\underline{\underline{A}}\,\underline{\underline{x}}(-t) = -\underline{\underline{A}}\,\underline{\underline{\tilde{x}}}(t)$$

i.e. $\underline{\underline{\tilde{x}}}' = -\underline{\underline{A}}\,\underline{\underline{\tilde{x}}}$

Sol'n to (2) same as (1) w/ reversed time

$$\underline{\underline{x}}(t) = c_1 e^{3t}\begin{bmatrix}1\\2\end{bmatrix} + c_2 e^{4t}\begin{bmatrix}2\\-3\end{bmatrix}, \quad \underline{\underline{x}}'(t) = 3c_1 e^{3t}\begin{bmatrix}1\\2\end{bmatrix} + 4c_2 e^{4t}\begin{bmatrix}2\\-3\end{bmatrix}$$



same as before, opposite arrows.

All trajectories receding from origin.

(nodal source)

<u>Terminology</u>: The origin is a <u>node</u> if

1. either every trajectory approaches 0 as $t \to \infty$ [sink] <u>or</u> every trajectory recedes (goes away) from the origin as time increases [source].

<u>AND</u> 2. Every trajectory is tangent to a straight line through the origin at the origin.