OH today 3-5 pm
tomorrow 4-6 pm
Midterm: 8-9 pm

1. Fourier series formulas.
2. 11a. sine series for $\cos(t)$ on $[0,\pi]$
3. Endpt problem.

1. **Bounds**

f 2L - periodic

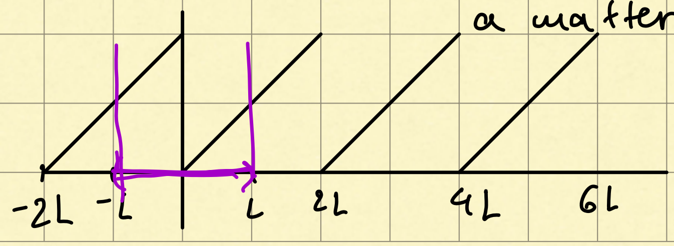
$$f \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{L}t\right) + b_n \sin\left(\frac{n\pi}{L}t\right) \right)$$

$$\underbrace{a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\left(\frac{n\pi}{L}t\right)dt} \qquad b_n = \ldots$$

2L periodic fct, can integrate over $[0,2L]$ and get same result:

$$a_n = \frac{1}{L}\int_0^{2L} f(t)\cos\left(\frac{n\pi}{L}t\right)dt$$

which one we choose is a matter of convenience.



$$a_0 = \frac{1}{L}\int_{-L}^{L} f(t)\,dt \qquad \text{hard.} \qquad \begin{array}{ll} f(t) = t & \text{for } t \in [0, L] \\ f(t) = t + 2L & \text{for } t \in [-L, 0) \end{array}$$

$$a_0 = \frac{1}{L}\left(\int_{-L}^{0} t + 2L\,dt + \int_{0}^{L} t\,dt \right)$$

$$= \dots$$

Easier:

$$\frac{1}{L}\int_{0}^{2L} f(t)\,dt = \frac{1}{L}\int_{0}^{2L} t\,dt$$

//

If f $2L$-periodic and even

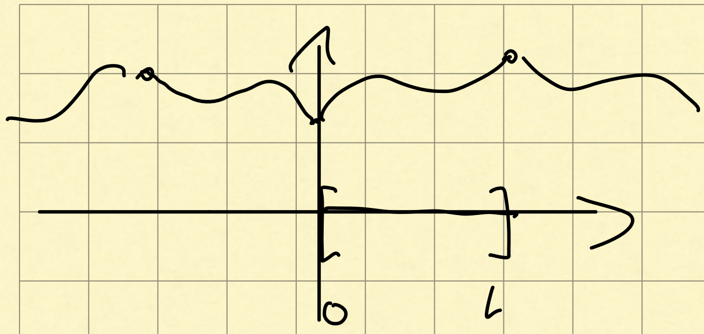
$$a_n = \frac{1}{L}\int_{-L}^{L} \underbrace{\underset{\text{even}}{f(t)} \underset{\text{even}}{\cos\left(\frac{n\pi}{L}t\right)}}_{\text{even}}\,dt$$

and $b_n = 0$

$$\rightarrow \quad a_n = \frac{1}{L}\cdot 2\int_{0}^{L} f(t)\cos\left(\frac{n\pi}{L}t\right)dt$$

this works for even f, not in general.

Extensions:



Sine series/odd extension

$$b_n = \frac{1}{L}\int_{-L}^{L} \underbrace{\underbrace{f_o(t)}_{\text{odd}}\ \underbrace{\sin\left(\frac{n\pi}{L}t\right)}_{\text{odd}}}_{\text{even}}\, dt$$

$$= \frac{2}{L}\int_0^L f_o(t)\sin\left(\frac{n\pi}{L}t\right)dt \quad //$$

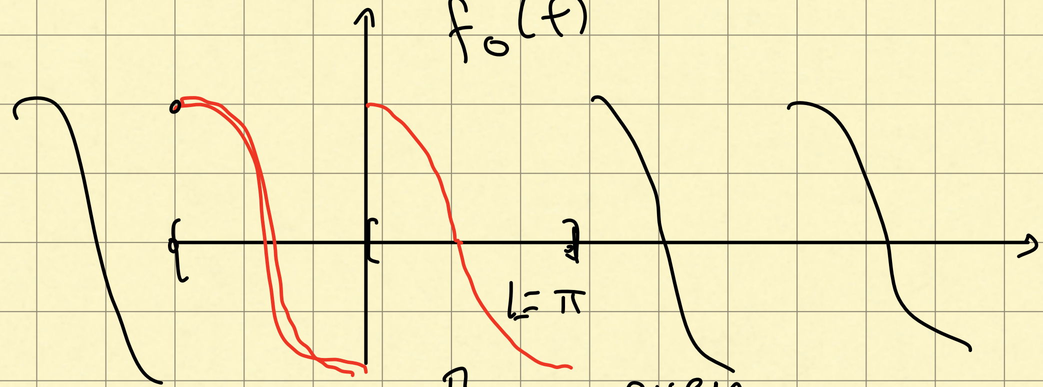
Review

11.a. Sine series for cosines.

$$f(t) = \cos(t) \quad \text{on} \quad [0,\pi]$$

$$f(t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\cancel{\pi}}{\cancel{\pi}}t\right)$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} \cos(t)\sin(nt)\,dt$$

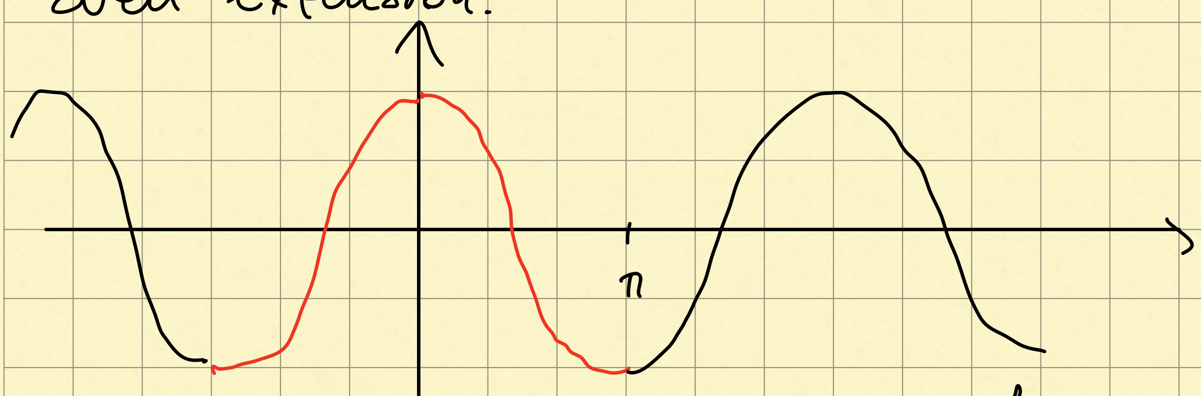


$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{f_o(t)\sin\left(\frac{n\pi}{\pi}t\right)}_{\text{even}} dt$$

$$= \frac{2}{\pi}\int_0^{\pi} f_o(t)\sin(nt)\,dt =$$

$$= \frac{2}{\pi}\int_0^{\pi} f(t)\sin(nt)\,dt.$$

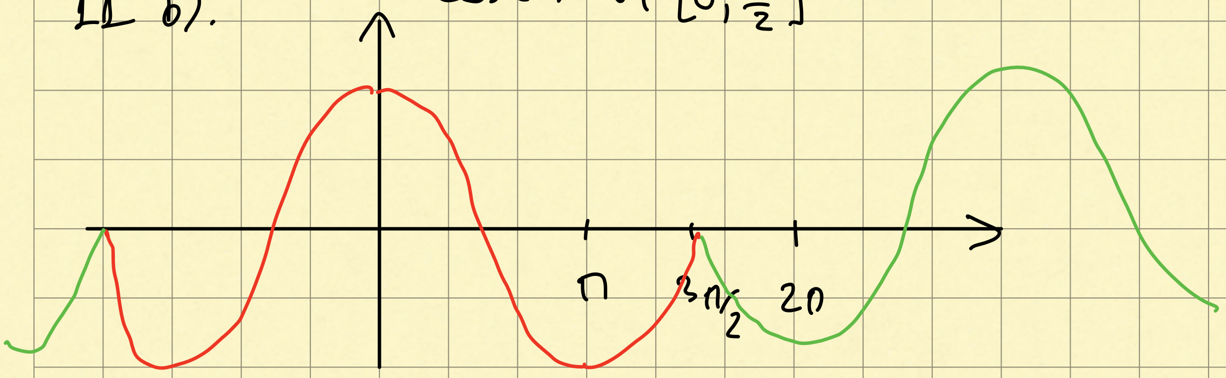
Even extension:



even extension of $\cos(t)$ on $[0,\pi]$ <u>is</u> the usual cosine.

11 b).

$\cos(t)$ on $[0, \frac{3\pi}{2}]$



Endpt problem

$$x'' + 4x = \underbrace{7t}_{} \qquad x'(0) = x'(\pi) = 0.$$

no x'

on $[0, \pi]$

Step 1: odd or even extension?

If no x' term in Dif eq.

If bdry condition is $x(0) = x(L) = 0$

$\rightarrow$ sine series

$x'(0) = x'(L) = 0$

$\rightarrow$ cosine series.

In this case: cosine series.

$$7t = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{\pi} t\right)$$

L

$$a_0 = \frac{2}{\pi}\int_0^{\pi} 7t\,dt$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} 7t\cos(nt)\,dt.$$

$$a_0 = \frac{2}{\pi} \frac{7t^2}{2}\Big|_0^{\pi} = 7\pi$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} 7t \cos(nt)\,dt = \frac{2}{\pi}\int_0^{\pi} 7t \frac{d}{dt}\left(\frac{\sin(nt)}{n}\right)dt$$

$$= \frac{2}{\pi} 7t \frac{\sin(nt)}{n}\Big|_0^{\pi} - \frac{2}{\pi}\int_0^{\pi} \frac{7}{n} \sin(nt)\,dt$$

$$= + \frac{14}{\pi n^2} \cos(nt)\Big|_0^{\pi} = + \frac{14}{\pi n^2}\left((-1)^n - 1\right)$$

Can Assume : bec. there are no x' term

$$x = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos(nt)$$

$$x'' = \sum_{n=1}^{\infty} (-n^2 A_n) \cos(nt)$$

So: plug in:

$$4\frac{A_0}{2} + \sum_{n=1}^{\infty} \left(-n^2 A_n + 4A_n\right)\cos(nt)$$

$$= \frac{a_0}{2} + \sum_{n=1} a_n \cos(nt)$$

for $n \neq 2$:

$$\begin{cases} A_n = \dfrac{1}{4-n^2}\, a_n = \dfrac{1}{4-n^2}\dfrac{14}{\pi n^2}\left((-1)^n - 1\right) \\ \\ A_0 = \dfrac{1}{4} \cdot a_0 = \dfrac{7\pi}{4}. \end{cases}$$

$$x = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos(nt)$$

formal sol'n in Fourier series form.