Vibrating string

$y_{tt} = a^2 y_{xx}$ $\quad 0 < x < L,\ t > 0$

$y(0,t) = y(L,t) = 0$

$y(x,0) = f(x)$

$y_t(x,0) = g(x)$

Split:

$$A:\begin{cases} y_{tt} = a^2 y_{xx} & (1) \\ y(0,t) = y(L,t) = 0 & (2) \\ y(x,0) = f(x) & (3) \\ y_t(x,0) = 0. & (4) \end{cases}$$

$$B:\begin{cases} y_{tt} = a^2 y_{xx} \\ y(0,t) = y(L,t) = 0 \\ y(x,0) = 0 \\ y_t(x,0) = g(x) \end{cases}$$

Find sol's $y_A, y_B \Rightarrow y = y_A + y_B$ solves original.

Solve Pr. A w/ sep. of variables.

Want: $y_A(x,t) = \sum_{j=1}^{\infty} c_j y_j(x,t)$

Here: y_j : building blocks

Want: $\begin{cases} \partial_t^2 y_j = a^2 \partial_x^2 y_j & (i) \\ y_j(0,t) = y(L,t) = 0 & (ii) \\ \partial_t y_j(x,0) = 0 & (iii) \end{cases}$