Also: $T'' = -\lambda a^2 T$

$$\Rightarrow \boxed{T'' = -a^2\left(\frac{n\pi}{L}\right)^2 T}$$

and

$$\frac{\partial y}{\partial t}(x,0) = 0 \Rightarrow X(x)T'(0) = 0$$

$$X \neq 0 \Rightarrow \boxed{T'(0) = 0}$$

So: solve

$$T'' + a^2\frac{n^2\pi^2}{L^2}T = 0$$

$$T'(0) = 0.$$

$$T(t) = A\cos\left(a\frac{n\pi}{L}t\right) + B\sin\left(a\frac{n\pi}{L}t\right)$$

$$T'(0) = 0 \Rightarrow B = 0$$

Building blocks:

$$y_n(x,t) = \underbrace{\sin\left(\frac{n\pi}{L}x\right)}_{X(x)}\underbrace{\cos\left(a\frac{n\pi}{L}t\right)}_{T(t)}$$

Soln:

$$y(x,t) = \sum_{n=1}^{\infty} A_n \cos\left(\frac{an\pi}{L}t\right)\sin\left(\frac{n\pi}{L}x\right)$$

Want:

$$y(x,0) = f(x)$$

$$\sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{L}x\right) = f(x)$$