Midterm 1: 6.30 - 7.30 RHPH 172

OH : 4.40 - 5.40 & tom. 3-5

7.1-7.2 due tomorrow.

Friday No Class.

1. Clockwise- C-Clockwise spirals
2. 9 practice problems
3. 3
4. Linearized $\to$ non-linear.
5. 7.2 $\begin{cases} x' + 2y' + x = 0 \\ x' - y' + y = 0 \end{cases}$ $x(0) = 0$, $y(0) = 1$

1. Ex: $\underline{x}' = \begin{bmatrix} 0 & -1 \\ 4 & 0 \end{bmatrix} \underline{x}$ (1)

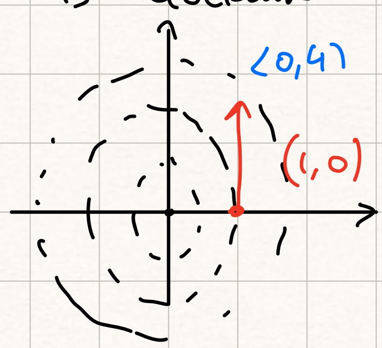
E-values:

$\lambda^2 + 4 = 0 \Rightarrow \lambda = \pm 2i$

$\to$ phase plane portrait

origin is a stable center.

Is clockwise or c-clockwise?



Vector corr. to (1,0)?

plug into (1) $\Rightarrow$ velocity vector $\left(\frac{dx}{dt}, \frac{dy}{dt}\right)\Big|_{(1,0)} = (0,4)$

2. Practice Problems ~~#9.~~ Similar Laplace tr. problem.

$$\begin{cases} x' + 2y' + x = 0 & x(0) = 0 \quad ① \\ x' - y' + y = 0 & y(0) = 1 \quad ② \end{cases}$$

$X(s) = \mathcal{L}\{x(t)\}, \quad Y(s) = \mathcal{L}\{y(t)\}$

① $\Rightarrow$

$$sX(s) - x(0) + 2(sY(s) - y(0)) + X(s) = 0$$

$$\to (s+1)X(s) + 2sY(s) = 2 \quad ③$$

② $\Rightarrow$ $sX(s) - x(0) - sY(s) + y(0) + Y(s) = 0$

$$\Rightarrow sX(s) - (s-1)Y(s) = -1 \quad ④$$

$$③ \cdot s \to s(s+1)X(s) + 2s^2 Y(s) = 2s$$

$$④ \cdot (s+1) \to s(s+1)X(s) - (s^2-1)Y(s) = -(s+1) \quad \ominus$$

$$(2s^2 + s^2 - 1)Y(s) = 2s + s + 1$$

$\Rightarrow Y(s) = \dfrac{3s+1}{3s^2-1}$ $\Rightarrow$ find $y(t)$ by partial fractions.

$$\frac{s + \frac{1}{3}}{s^2 - \frac{1}{3}} = \frac{s + \frac{1}{3}}{\left(s - \sqrt{\frac{1}{3}}\right)\left(s + \frac{1}{\sqrt{3}}\right)}$$

$$= \frac{A}{s - \sqrt{\frac{1}{3}}} + \frac{B}{s + \sqrt{\frac{1}{3}}} \ldots$$

find $y(t)$ then:

for x $\quad x' - y' + y = 0 \Rightarrow x' = \underbrace{y' - y}_{\text{known}}$

integrate to find x //

3 (practice problems).

$$A = \begin{bmatrix} -15 & -7 & 4 \\ 34 & 16 & -11 \\ 17 & 7 & 5 \end{bmatrix}, \text{ solve } x' = Ax$$

$A \to$ e-value $\lambda = 2$ w/ mult. 3.

Q: Is λ defective? If yes, what is the defect?

Solve the eigenvector system.

$$(A - 2I)v = 0.$$

$$\begin{bmatrix} -17 & -7 & 4 \\ 34 & 14 & -11 \\ 17 & 7 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

row red.

2·① + ② → ②

① + ③ → ③

$$\begin{bmatrix} -17 & -7 & 4 \\ 0 & 0 & -3 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad v_3 = 0 \quad \& \quad -17v_1 - 7v_2 + 4v_3 = 0$

$\Rightarrow \quad v_2 = -\frac{17}{7} v_1$

e-vector $\begin{bmatrix} v_1 \\ -\frac{17}{7} v_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -\frac{17}{7} \\ 0 \end{bmatrix} v_1$

can't find 2 or more lin. indep. e-vectors $\Rightarrow$ defect $= 3 - 1 = 2$.

multiplicity

All this was to find <u>defect</u>.

Look for chain of gen. e-vectors of length defect + 1 = 3.

Start at top: find gen. e-vector of rank 3

$$(A - 2I)^3 \underline{\underline{v}}_3 = \underline{\underline{0}} \quad (1)$$
$$(A - 2I)^2 \underline{\underline{v}}_3 \neq \underline{\underline{0}} \quad (2)$$

Given: $(\underline{\underline{A}} - 2\underline{\underline{I}})^3 = \underline{\underline{0}} \Rightarrow (1)$ says nothing

$$(A - 2I)^2 = \begin{bmatrix} 119 & 49 & 21 \\ -289 & -119 & -51 \\ 0 & 0 & 0 \end{bmatrix}$$

Want: $\begin{bmatrix} 119 & 49 & 21 \\ -289 & -119 & -51 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Cands $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}$

take $\underline{\underline{v}}_3 = \begin{bmatrix}1\\0\\0\end{bmatrix}$.

$$\underline{\underline{v}}_2 = (A - 2I)\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$= \begin{bmatrix}-17 & -7 & 4\\ 34 & 14 & -11\\ 17 & 7 & 3\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$= \begin{bmatrix}-17\\34\\17\end{bmatrix}$$

ber $\underline{\underline{v}}_2 = (A-2I)\underline{\underline{v}}_3$

$$\underline{\underline{v}}_1 = (A-2I)\underline{\underline{v}}_2 = (A-2I)^2\underline{\underline{v}}_3$$

$$= \begin{bmatrix}119 & 49 & 21\\ -289 & -119 & -51\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$= \begin{bmatrix}119\\-289\\0\end{bmatrix}.$$

<u>Chain:</u> $v_3 = \begin{bmatrix}1\\0\\0\end{bmatrix}, \underline{\underline{v}}_2 = \begin{bmatrix}-17\\34\\17\end{bmatrix}, \underline{\underline{v}}_1 = \overbrace{\begin{bmatrix}119\\-289\\0\end{bmatrix}}^{\text{true e-vector.}}$

Is it $-289 = -\frac{17}{7}\, 119$ ✓

important that vectors
in chain are tied by rule

$$v_k = (A - \lambda I)\, v_{k+1}$$