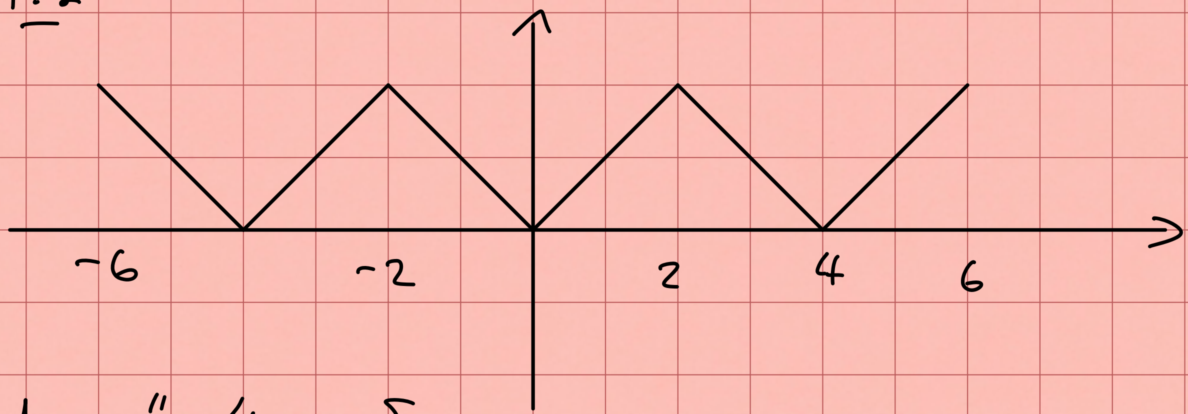
P. 1



1. $x'' + 4x = F$

F even, no sine terms.

$$F = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{2} t\right)$$

$$a_0 = \frac{2}{2} \int_0^2 2t \, dt = 2 \frac{t^2}{2} \Big|_0^2 = 4$$

$$a_n = \frac{2}{2} \int_0^2 2t \cos\left(\frac{n\pi}{2} t\right) dt$$

$$= \frac{4}{n\pi} \int_0^2 t \frac{d}{dt}\left(\sin\left(\frac{n\pi}{2} t\right)\right) dt$$

$$= \frac{4}{n\pi}\left(t \sin\left(\frac{n\pi}{2} t\right)\right)\Big|_0^2 - \frac{4}{n\pi} \int_0^2 \sin\left(\frac{n\pi}{2} t\right) dt$$

$$= \frac{8}{(n\pi)^2} \cos\left(\frac{n\pi}{2} t\right)\Big|_0^2 = \frac{8}{(n\pi)^2}\left((-1)^n - 1\right)$$

3. Can assume that x_{sp} only has cosine terms bec. $x''+4x$ has no x' term and F only has cosine terms.

$$x_{sp} = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{2}t\right)$$

$$x_{sp}'' = \sum_{n=1}^{\infty} \left(-\left(\frac{n\pi}{2}\right)^2 A_n\right) \cos\left(\frac{n\pi}{2}t\right)$$

So $x_{sp}'' + 4x_{sp} = F \Rightarrow$

$$\rightarrow \quad 4\frac{A_0}{2} = \frac{a_0}{2} = 2 \Rightarrow A_0 = 1$$

$$-\left(\frac{n\pi}{2}\right)^2 A_n + 4A_n = \frac{8}{(n\pi)^2}\left((-1)^n - 1\right)$$

$$\Rightarrow A_n = \frac{8}{4-\left(\frac{n\pi}{2}\right)^2} \frac{(-1)^n - 1}{(n\pi)^2}$$

4. gen. sol'n:

$$A\cos(2t) + B\sin(2t) + \frac{1}{2} + \sum_{n=1}^{\infty} \frac{8}{4-\left(\frac{n\pi}{2}\right)^2} \frac{(-1)^n - 1}{(n\pi)^2}$$

P. 2 $\quad x'' + \frac{\pi^2}{4}x = F(t)$

Write $x = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{2}t\right)$

$$x'' + \frac{\pi^2}{4}x$$

$$= \frac{A_0}{2}\cdot\frac{\pi^2}{4} + \sum_{n=1}^{\infty}\underbrace{\left(-\frac{n^2\pi^2}{4} + \frac{\pi^2}{4}\right)}_{\text{this expression vanishes when } n=1.} A_n \cos\left(\frac{n\pi}{2}t\right).$$

Setting equal to F

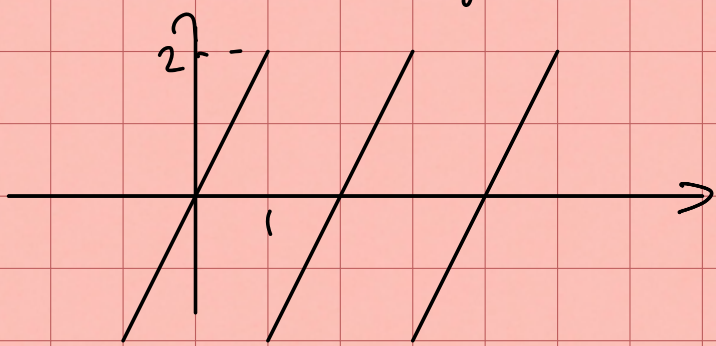
$$\frac{A_0}{2}\cdot\frac{\pi^2}{4} = 2 \Rightarrow A_0 = \frac{1}{\pi^2}$$

$$0 \cdot A_1 = -\frac{16}{\pi^2} \qquad \text{impossible.}$$

So A_1 can't be found, meaning that our assumption that x_{sp} had a sol'n w/ a F.S. expansion was incorrect.

P.3

1. F(t) has graph as follow



$$F(t) = \sum_{n=1}^{\infty} b_n \sin(n\pi t)$$

For $n = 2$ we have $2\pi = \sqrt{\frac{k}{m}}$ so there is pure resonance.

2. $\sqrt{\frac{k}{m}} = \sqrt{5}$. since $\frac{n\pi}{1} \neq \sqrt{5}$ for all n there is no resonance.

4. Find expansion of F in Problem 2.b

$$F = \sum_{n=1}^{\infty} b_n \sin(n\pi t)$$

$$b_n = 2\int_0^1 1 \cdot \sin(n\pi t)\,dt = \frac{2\cos(n\pi t)}{n\pi}\Big|_0^1$$

$= \frac{2}{n\pi}\left((-1)^n - 1\right)$

$b_0 = 0, \; b_1 = \frac{-4}{\pi}, \; b_2 = 0, \; b_3 = \frac{-4}{3\pi},$

$b_4 = 0, \; b_5 = \frac{-4}{5\pi}$

$n = 1:$

$\omega_1 = \pi$

$\tan \alpha_1 = \frac{0.01\pi}{4 - 2\pi^2} \approx -0.013$

$\Rightarrow \alpha_1 = \pi + \arctan(\alpha_1)$

≈ 3.127

$n = 3$: Similarly, w/ software.

$\alpha_3 \approx 3.13524$

$n = 5:$

$a_5 \approx 3.13586$

Then plug in for coef. of x_{sp}

$$x_{sp} = 0.32 \sin(\pi t - 3.127) + 0.032 \sin(3\pi t - 3.13524) + 0.0115 \sin(5\pi t - 3.13586)$$