Ex 1.

1. The signs indicate that x stands for rabbits & y for foxes.

2. $200x - 4xy = 0 \Rightarrow x(200-4y) = 0 \Rightarrow$
$x = 0$ or $y = 50$

$-150y + 2xy = 0 \Rightarrow y(-150+2x) = 0$
$\Rightarrow y = 0$ or $x = 75$

So $(0,0)$ and $(75,50)$

3.

$$J(x,y) = \begin{bmatrix} 200-4y & -4x \\ 2y & -150+2x \end{bmatrix},$$ so:

At $(0,0)$:

$$\underline{\underline{u}}' = \begin{bmatrix} 200 & 0 \\ 0 & -150 \end{bmatrix} \underline{\underline{u}}$$

Eigenvalues: $\lambda_1 = -200$, $\lambda_2 = 150$ so we have an (unstable) saddle.

At $(75,50)$:

$$\underline{\underline{u}}' = \begin{bmatrix} 0 & -300 \\ 100 & 0 \end{bmatrix} \underline{\underline{u}}.$$

Eigenvalues: $\lambda^2 + 30000 = 0$ $\lambda = \pm i\, 100\sqrt{3}$
origin is a stable (but not asymptotically stable) center.

4. We have unstable saddle at $(0,0)$. At $(75,50)$

we have either stable center, or as. stable spiral sink, or unstable spiral source.

5. Option B.

6. The populations evolve periodically:
pop. of rabbits will decrease, then reach a min, increase, reach a max, decrease again etc. Similarly for foxes, except they initially increase.
If initially we have 75 rabbits & 50 foxes, the populations will stay constant (equilibrium sol'n)
//.

Logistic Populations.
i. $c_1, c_2 > 0$: competition
ii. opposite signs: predation
iii. $c_1, c_2 < 0$: cooperation

Ex. 2

1. Competition.
2. $60 \cdot 6 - 4 \cdot 36 - 3 \cdot 6 \cdot 12 =$
$= 360 - 144 - 216 = 0$ ✓
$42 \cdot 12 - 2 \cdot 144 - 3 \cdot 6 \cdot 12 = 0$ ✓

Compute linearization:

$$J = \begin{bmatrix} 60 - 8x - 3y & -3x \\ -3y & 42 - 4y - 3x \end{bmatrix}$$

so

$$\underline{\underline{u}}' = \begin{bmatrix} -24 & -18 \\ -36 & -24 \end{bmatrix}$$

Eigenvalues: $(\lambda + 24)^2 - 648 = 0$

$\Rightarrow \lambda_1 \simeq 1.46, \quad \lambda_2 \simeq -49.46$

so we have an unstable saddle for both the linearized and the non-linear system.

3. In the first case, the population $y(t)$ becomes extinct as $t \to \infty$, in the second x becomes extinct. So even one individual can tip the balance enough to change completely the final outcome.