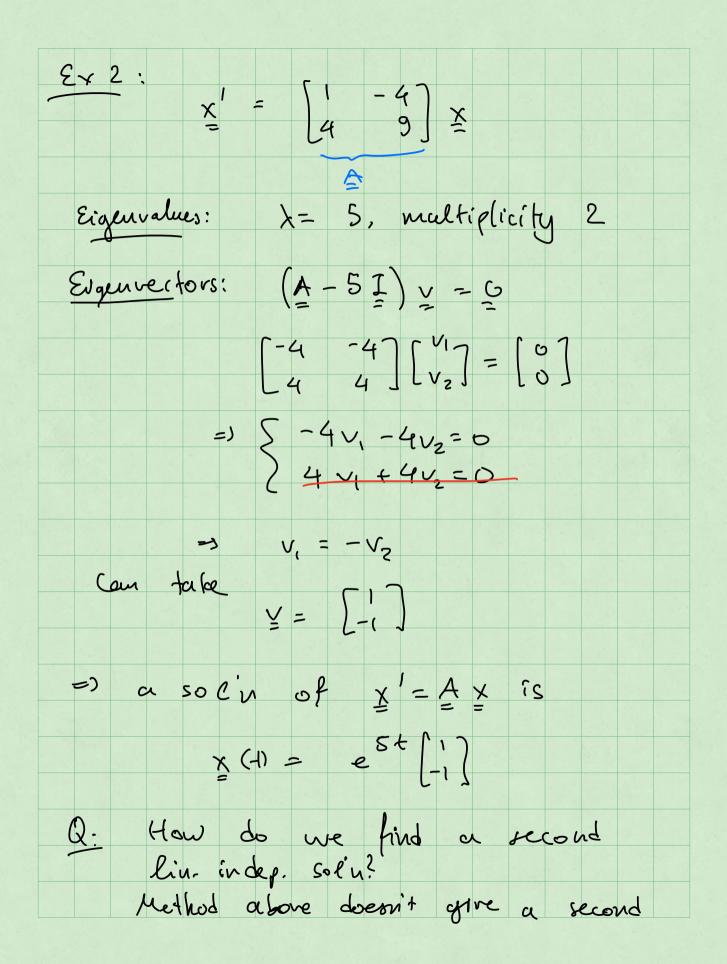
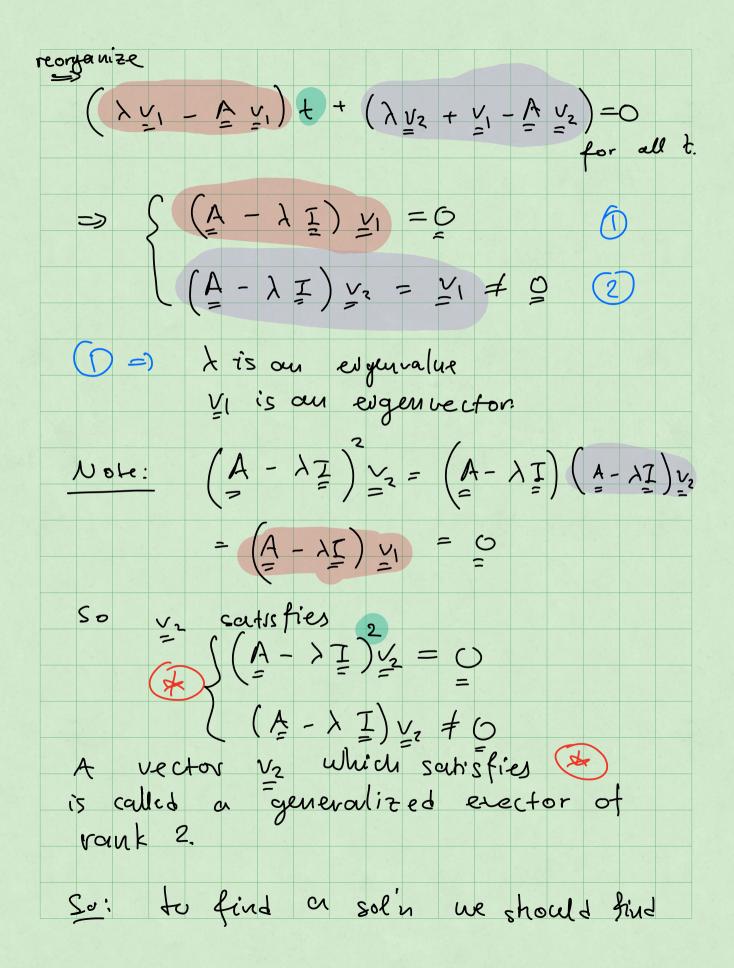
Lesson 6
Ol/24/2022
SSS Linear systems with defective eagenvalues

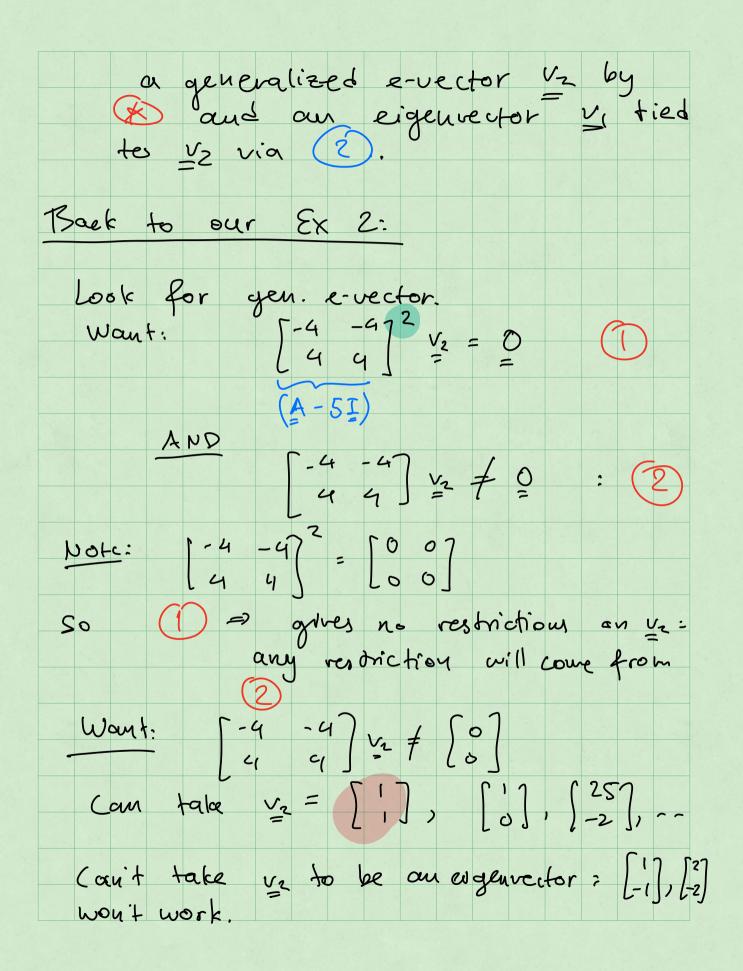
$$E \times I : I' = A \times , A = [I 0]$$

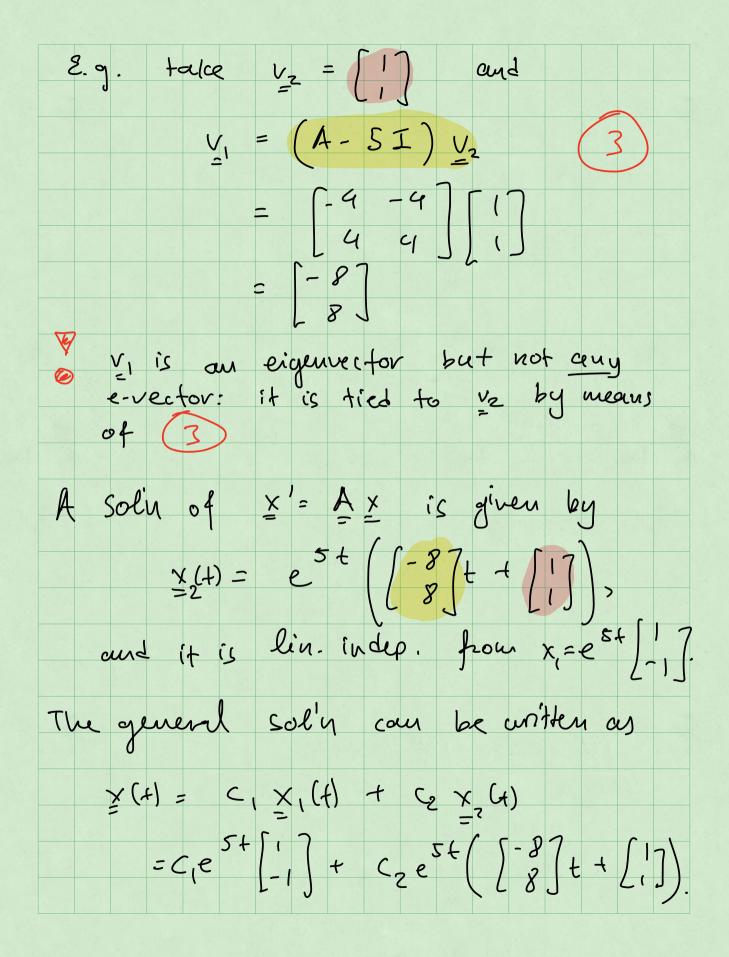
Womt: 2 lin. indep. sols.
Find evalues:
 $det (A - \lambda I) = I 0 I - \lambda I$
 $e = (I - \lambda)^2$
 $e = (I - \lambda)^2 = 0 \Rightarrow \lambda = 1$ multiplicity 2.
Seek eigenvectors
 $(A - 1 - I) \vee = 0 \Rightarrow 0 \vee = 0$
Any non-zero vector is an evector.
In perficular, we can find 2 lin indep.
 $e \cdot vectors.$
 $E_{I}: V_{I} = [0], V_{e} = [0]$
 $\Rightarrow We can built 2 lin. indep. sols as
 $x_{1}(I) = e^{I} [0], x_{2}(I) = e^{I} [0]$
 $\Rightarrow Qui - sol n: x_{1} = c_{1} e^{I} [0] + c_{2} e^{I} [0]$$



lin. indep. soln. A: Look for solin in a different form. Terminology! If & is an eigenvalue of A w/ multiplicity k and we can find at most p = k lin. indep. eigenvectors associated w/ & then we say that I hay $\frac{1}{1} \frac{1}{2 \times 1} = \frac{1}{2 \times 2}$ defect If defect 71 then h is called defective To deal ul sigenvalues of defect 1. Assume that a solin of x'= Ax has the form $x(t) = e^{\lambda t} \left(\frac{y}{z} t + \frac{y}{z} \right)$ w/ v, + 9 Plug in: $e^{\lambda t} = 0$ $he^{\lambda t} \left(v_1 + v_2 \right) + e^{\lambda t} v_1 = A \left(e^{\lambda t} \left(v_1 + v_2 \right) \right)$







Again:
$$X(t) = e^{St} \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} t + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$$
 is
unit a soly for $X' = A \times$
even though $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is a generalized
evector and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an e-vector
bec. $\begin{bmatrix} -1 \\ 1 \end{bmatrix} \neq \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.
Summary of method for systems of
an e-veloe of defect L .
1. Find eigenvalue(s) and corresponding
e-vectors
2. For a defective eigenvalue λ ,
find a generalized e-vector of
rank 2 , i.e. solve
 $\left(\begin{bmatrix} A & -\lambda \\ 1 \end{bmatrix} \frac{V_2}{2} = 0 \quad \frac{V_2}{2} \neq 0 \\ \begin{bmatrix} (A & -\lambda \\ 1 \end{bmatrix} \frac{V_2}{2} \neq 0 \\ \end{bmatrix}$
Set $V_1 = e^{\lambda t} (V_1 + V_2)$ is a solin.
 $e^{\lambda t} v_1$ is also a solin, lin.
independent.