# Quiz 2

Let $E$ be the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 2$, and $f(x, y, z)$ be a continuous function on it. Set up an integral $\iiint_E f(x, y, z)dV$ **in the order** $dxdzdy$.

Eqns: $x = \sqrt{z - y^2}$

$x = -\sqrt{z - y^2}$

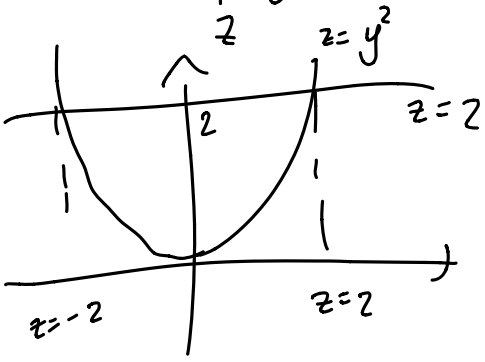
$z = 2$

$-\sqrt{z-y^2} \le z \le \sqrt{z-y^2}$

Find projection (eliminate $x$) $\sqrt{z-y^2} = -\sqrt{z-y^2}$

$\Rightarrow z = y^2 \Rightarrow y^2 \le z \le 2$

$y^2 = 2 \Rightarrow y = \pm\sqrt{2} \Rightarrow -\sqrt{2} \le y \le \sqrt{2}$



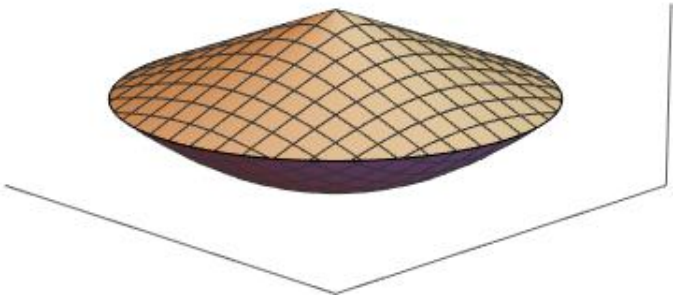

So:

$$\iiint_E f\,dV = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{y^2}^{2} \int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} f(x,y,z)\,dx\,dz\,dy$$

Let $E$ be the solid of the picture below, bounded below by the paraboloid $z = 4x^2 + 4y^2$ and bounded above by the cone $z = 8 - 4\sqrt{x^2 + y^2}$. Set up but do not evaluate an integral that computes the volume of $E$.

Use cylindrical coordinates

(*) $z = 4r^2$, $z = 8 - 4r$ $\Rightarrow 4r^2 \le z \le 8 - 4r$



Projection: intersect (*)

$4r^2 = 8 - 4r$

$\Rightarrow 4r^2 + 4r - 8 = 0 \Rightarrow$

$r = -2$ or $\boxed{r = 1}$

(irrelevant)

So $0 \le \theta \le 2\pi$, $0 \le r \le 1$

$$V = \int_0^{2\pi} \int_0^1 \int_{4r^2}^{8-4r} 1 \cdot r\,dz\,dr\,d\theta$$