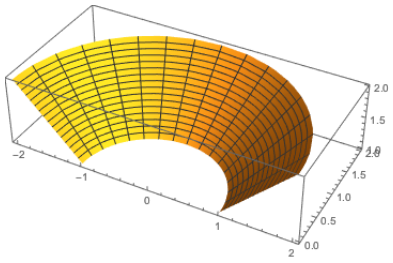
Quiz 6

Let S be the surface that consists of a part of the cone $z^2 = x^2 + y^2$ that lies between the planes z=1 and z=2 and it satisfies $y \geq 0$. Find a parametrization for S and use it to compute the surface integral

$$\iint_S y dS.$$



Write

$\vec{r}(u,v) = \langle u, v, \sqrt{u^2+v^2} \rangle$, $v \geq 0$, $1 \leq \sqrt{u^2+v^2} \leq 2$

$\vec{r}_u \times \vec{r}_v = \left\langle -\frac{1}{2\sqrt{u^2+v^2}} \cdot 2u, -\frac{1}{2\sqrt{u^2+v^2}} \cdot 2v, 1 \right\rangle$

$\Rightarrow |\vec{r}_u \times \vec{r}_v| = \sqrt{\frac{u^2}{u^2+v^2} + \frac{v^2}{u^2+v^2} + 1} = \sqrt{2}$

So $\iint_S y\, dS = \iint_{\substack{v \geq 0 \\ 1 \leq \sqrt{u^2+v^2} \leq 2}} v\sqrt{2}\, dA \overset{\text{polar}}{=} \int_0^{\pi} \int_1^2 r \sin\theta \cdot \sqrt{2}\, r\, dr\, d\theta$

$= \sqrt{2} \cdot 2 \left. \frac{r^3}{3} \right|_1^2 =$

$= 2\sqrt{2} \cdot \frac{7}{3}$