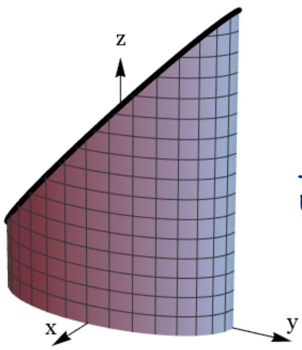
Let S be the surface consisting of the part of the generalized cylinder $x^2 + \dfrac{y^2}{4} = 1$, between the planes $z = 0$ and $z = y + 3$, that also satisfies $x > 0$. We give S orientation towards the positive x axis (this means that the x coordinate of the unit normal vector field has to be always positive).

(i) Compute $\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F}(x, y, z) = \langle y, x, z \rangle$.

$\vec{r}(u,v) = \langle \cos u, 2\sin u, v \rangle$ $\quad -\frac{\pi}{2} \le u \le \frac{\pi}{2}$

$0 \le v \le 2\sin u + 3$



$\vec{r}_u = \langle -\sin u, 2\cos u, 0 \rangle$

$\vec{r}_v = \langle 0, 0, 1 \rangle$

$\vec{r}_u \times \vec{r}_v = \langle 2\cos u, \sin u, 0 \rangle$ correct orientation

$$\iint_S \vec{F} \cdot d\vec{S} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\sin u + 3} \langle 2\sin u, \cos u, v \rangle \cdot \langle 2\cos u, \sin u, 0 \rangle \, dv\, du$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2\sin u + 3)\, 5 \sin u \cos u \, du = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 10 \sin^2 u \cos u + 15 \sin u \cos u \, du$$

$$= \frac{10}{3} \sin^3 u + \frac{15}{2} \sin^2 u \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{10}{3} \cdot 2 = \frac{20}{3}$$