Worksheet

April 20, 2020

1. (Triple integral) Set up an integral $\iiint_E f(x, y, z)dV$, where E is bounded above by the plane $z = 3 - x - y$, below by the xy plane, and also bounded by the planes $x = -1$, $x = 1$, $y = 0$ and $y = 1$ **in the order** $dxdzdy$.

2. (Spherical or cylindrical coordinates) Compute the volume of the bounded domain lying between the cones $z^2 = x^2 + y^2$ and $z^2 = 3(x^2 + y^2)$, and under the plane $z = 3$.

3. (Change of variables) If a transformation T is written as $x = x(u, v)$ and $y = y(u, v)$ and given a point (u_0, v_0) on the uv plane, we can produce an affine transformation dT called the **differential** or **pushforward** of T at (u_0, v_0) that is the best affine approximation of T near (u_0, v_0) and is given by

$$x = \frac{\partial x}{\partial u}_{|(u,v)=(u_0,v_0)}(u - u_0) + \frac{\partial x}{\partial v}_{|(u,v)=(u_0,v_0)}(v - v_0) + x(u_0, v_0)$$
$$y = \frac{\partial y}{\partial u}_{|(u,v)=(u_0,v_0)}(u - u_0) + \frac{\partial y}{\partial v}_{|(u,v)=(u_0,v_0)}(v - v_0) + y(u_0, v_0).$$

   For the transformation $T(u, v) = (\frac{u^2}{v}, u^2 v)$ defined on $\{(u, v) : u > 0, v > 0\}$:

   (a) Find the transformation dT relative to the point (1,1).

   (b) Find and draw the image of the box $[1, 2] \times [1, 2]$ under T and dT.

4. (Chain Rule) A fly flies in a room along the curve

$$c(t) = (2\sin(t), \cos(t), 2t),$$

   where t is measured in seconds. The temperature at the point (x, y, z) of the room is given by the function

$$T(x, y, z) = x^2 + 2e^z y,$$

in ° C. As the fly moves, it experiences a different temperature at each point. Find the instantaneous rate of change of the temperature that the fly experiences at time π seconds (include units).

5. (Change of Variables) Make a change of variables to evaluate the integral

$$\iint_R e^{\frac{y-x}{y+x}}\, dA,$$

where R is the trapezoidal region with vertices (1,0), (2,0), (0,2) and (0,1).

6. *(Chain Rule, an interesting conceptual question) What's wrong with the following argument? Suppose we are given a function $w = f(x, y, z)$, where $z = g(x, y)$. Then, wishing to compute $\frac{\partial w}{\partial x}$, we draw a tree diagram and find that at each point (x, y) we have

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial x} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial x} \implies \frac{\partial w}{\partial z}\frac{\partial z}{\partial x} = 0, \tag{1}$$

so at each point either $\frac{\partial w}{\partial z} = 0$ or $\frac{\partial z}{\partial x} = 0$, so something is wrong: there were no assumptions on w or z!

Hint: it might help you to work out a specific example.