Homework Set 2

Due: Wednesday July 6th

Section 15.5

Exercise: Find the mass and center of mass of the lamina that occupies the region D and has the given density function ρ: D is bounded by $y = \sqrt{x}$, $y = 0$, and $x = 1$; $\rho(x, y) = 9x$. (answer: $\frac{18}{5}$, $(\frac{5}{7}, \frac{5}{12})$).

Section 15.7

13: Calculate the triple integral: $\iiint_E 6xy dV$, where E lies under the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}, y = 0$, and $x = 1$. (answer: $\frac{65}{28}$)

17: Calculate the triple integral: $\iiint_E x dV$, where E is bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$. (answer: $\frac{16\pi}{3}$)

19: Use a triple integral to find the volume of the tetrahedron bounded by the coordinate planes and the plane $2x + y + z = 4$. (answer: $\frac{16}{3}$)

34: (look at picture in book) The figure shows the region of integration for the integral

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x, y, z) dy dz dx.$$

Rewrite this integral as an equivalent iterated integral in the five other orders (you don't need to show work for this problem).*

Section 15.8

Exercise: Use cylindrical coordinates to calculate the integral $\iiint_E e^z dV$, where E is enclosed by the paraboloid $z = 10 + x^2 + y^2$, the cylinder $x^2 + y^2 = 4$ and the xy-plane. (answer: $(-4 - e^{10} + e^{14})\pi$)

22: Use cylindrical coordinates to find the volume of the solid that lies within both the cylinder $x^2+y^2 = 16$ and the sphere $x^2 + y^2 + z^2 = 36$. (answer: $\frac{4}{3}(216 - 40\sqrt{5})\pi$)

30: Evaluate the integral by changing to cylindrical coordinates:

$$\int_{-4}^{4}\int_{0}^{\sqrt{16-x^2}}\int_{0}^{16-x^2-y^2}\sqrt{x^2+y^2}\,dzdydx.$$

(answer: $\frac{2048\pi}{15}$)

Section 15.9

32: Use spherical coordinates. Let H be a solid hemisphere of radius 2 whose density at any point is proportional to its distance from the center of the base. (Let K be the constant of proportionality.) Find the center of mass of H. (Assume the upper hemisphere of a sphere centered at the origin.) (answer: $(0,0,\frac{4}{5})$)

39: Evaluate the integral by changing to spherical coordinates:

$$\int_{0}^{6}\int_{0}^{\sqrt{36-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{72-x^2-y^2}} xy\,dzdydx.$$

(answer: $\frac{1296}{5}\sqrt{2}(8-5\sqrt{2})$)

* This is an excellent exercise to practice integrating in different orders. To check if your answers are correct, after you've set up the integral, you can experiment by using various functions f and check that the result of the integration is always the same.