

Homework Set 3

Due: Monday July 18th

Section 15.10

6: Find the Jacobian of the transformation:

$$x = 6v + 6w^2, y = 4w + 4u^2, z = 2u + 2v^2.$$

24: Evaluate the integral by making an appropriate change of variables.

$$\iint_R 5(x+y)e^{x^2-y^2} dA,$$

where R is the rectangle enclosed by the lines $x - y = 0$, $x - y = 7$, $x + y = 0$, and $x + y = 9$. (answer: $\frac{5}{14}(e^{63} - 64)$)

26: Evaluate the integral by making an appropriate change of variables: $\iint_R 6 \sin(9x^2 + 4y^2) dA$, where R is the region in the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$. (answer: $\frac{1}{4}\pi(1 - \cos(1))$)

14.5

1: Use the chain rule to find dz/dt , where $z = x^2 + y^2 + xy$, $x = \sin(t)$, $y = 8e^t$.

13: If $z = f(x, y)$, where f is differentiable, and $x = g(t)$, $g(6) = -7$, $g'(6) = -8$, $y = h(t)$, $h(6) = 7$, $h'(6) = -2$, $f_x(-7, 7) = 1$, $f_y(-7, 7) = -9$ find dz/dt when $t = 6$.

38: The radius of a right circular cone (this means that its axis is perpendicular to its base) is increasing at a rate of 1.7 in/s while its height is decreasing at a rate of 2.2 in/s . At what rate is the volume of the cone changing when the radius is 150 in. and the height is 125 in. ? (answer: $4750\pi \text{ in}^3/\text{s}$)

49: Show that any function of the form $z = f(x + at) + g(x - at)$ is a solution to the wave equation $\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}$ (that is, show that any z of this form satisfies the equation).

Hint: Let $u = x + at$, $v = x - at$.

14.6

8: Consider the following: The function $f(x, y) = y^3/x$, the point $P(1, 2)$, and the vector $\vec{u} = \frac{1}{3}(2\vec{i} + \sqrt{5}\vec{j})$. Find

- (a) Find the gradient of f .
- (b) Evaluate the gradient at the point P .
- (c) Find the rate of change of f at P in the direction of the vector \vec{u} .

34: Suppose you are climbing a hill whose shape is given by the equation $z = 1000 - 0.005x^2 - 0.01y^2$, where x , y and z are measured in meters, and you are standing at a point with coordinates $(60, 40, 966)$. The positive x -axis points east and the positive y -axis points north.

- (a) If you walk due south, will you start to ascend or descend?
- (b) If you walk northwest, will you start to ascend or descend?
- (c) In which direction is the slope largest? What is the rate of ascent in that direction? At what angle above the horizontal does the path in that direction begin?

Exercise I: Find the maximum rate of change of f at the given point and the direction in which it occurs.

- (a) $f(x, y) = 12x^3 + \cos(2x + 4)$, $(2, 3)$
- (b) $f(x, y, z) = \tan(9x + 2y + 3z)$, $(1, -9, 3)$