Homework Set 4

Due: Monday July 24th

Do for yourself: (you don't need to turn in this problem) Exercises 11-14 from section 16.1 (match the vector fields with their plots). You can check your answers using the Mathematica file I sent you, or any other type of software capable of plotting vector fields.

Turn in the following problems:

16.1

Exercise: Find the gradient vector field of f, where $f(x, y, z) = x\cos(\frac{4y}{z})$

16.2

13: Evaluate the line integral, where C is the given curve: $\int_C xye^{yz}dy$, $C : x = t$, $y = t^2$, $z = t^3$, $0 \le t \le 1$. (Answer: $\frac{2}{5}(e-1)$)

19: Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is given by the vector function $\mathbf{r}(t)$. $\mathbf{F}(x, y) = xy\mathbf{i} + 6y^2\mathbf{j}$, $\mathbf{r}(t) = 16t^6\mathbf{i} + t^4\mathbf{j}$, $0 \le t \le 1$. (answer: 98)

9: Evaluate the integral $\int_C xyzds$, where $C : x = 2\sin t, y = t, z = -2\cos t, 0 \le t \le \pi$.

42: The force exerted by an electric charge at the origin on a charged particle at a point (x, y, z) with position vector $\mathbf{r} = \langle x, y, z\rangle$ is $\mathbf{F}(\mathbf{r}) = K\mathbf{r}/|\mathbf{r}|^3$ where K is a constant. Find the work done as the particle moves along a straight line from (2, 0, 0) to (2, 4, 5). (Answer: $K(\frac{1}{2} - \frac{1}{3\sqrt{5}})$).

48: The base of a circular fence with radius 10 m is given by $x = 10\cos t$, $y = 10\sin t$. The height of the fence at position (x, y) is given by the function $h(x, y) = 5 + 0.05(x^2 - y^2)$, so the height varies from 0 m to 10 m. Suppose that 1 L of paint covers 100 m^2. Determine how much paint you will need if you paint both sides of the fence. (Answer: 2π)

Exercise I Evaluate the line integral $\int_C xe^y dx$, where C is the arc of the curve $x = e^y$ from (1,0) to $(e^5, 5)$.

16.3

6: Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$: $\mathbf{F}(x, y) = (3x^2 - 3y^2)\mathbf{i} + (6xy + 5)\mathbf{j}$.

8: Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$: $\mathbf{F}(x, y) = (ye^x + \sin(y))\mathbf{i} + (e^x + x\cos(y))\mathbf{j}$.

Exercise I If $\mathbf{F}(x, y, z) = 3y^2\mathbf{i} + (6xy + 3e^{3z})\mathbf{j} + 9ye^{3z}\mathbf{k}$, find a function f such that $\nabla f = F$ (assume the vector field is conservative).