# Homework Set 5

Due: Wednesday August 3rd

**Turn in the following problems:**

## 16.3

**12:** Find a function $f$ such that $\mathbf{F} = \nabla f$ and use it to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the curve $C$. Here, $\mathbf{F}(x, y) = x^2\mathbf{i} + y^2\mathbf{j}$ and $C$ is the arc of the parabola $y = 2x^2$ from (-1,2) to (2,8).

## 16.4

**11:** Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ (Check the orientation of the curve before applying the theorem): $\mathbf{F}(x, y) = \langle y\cos x - xy\sin x, xy + x\cos x\rangle$, $C$ is the triangle from (0, 0) to (0, 4) to (2, 0) to (0, 0). (answer: $\frac{-16}{3}$)

**6:** Use Green's theorem to evaluate the line integral $\int_C \cos(y)dx + x^2\sin(y)dy$ along the given positively oriented curve, where $C$ is the rectangle with vertices (0,0), (1,0), (1,3), (0,3). (answer: $2(1 - \cos(3))$).

**19:** Use one of the formulas below to find the area under one arch of the cycloid $x(t) = t - \sin(t), y = 1 - \cos(t)$ :

$$A = \oint_C xdy = -\oint_C ydx = \frac{1}{2}\oint_C xdy - ydx.$$

(answer: $3\pi$)

**28:** Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle x^2 + y, 3x - y^2\rangle$ and $C$ is the positively oriented boundary curve of a region $D$ that has area 6.

## 16.5

**3:** Find the curl and divergence of the vector field $\mathbf{F}(x, y, z) = 7xye^z\mathbf{i} + yze^x\mathbf{k}$.

**5:** Find the curl and divergence of the vector field $\mathbf{F}(x, y, z) = \dfrac{4}{\sqrt{x^2 + y^2 + z^2}}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$.

**12:** Let $\mathbf{F}$ be a vector field and $f$ a scalar field. For each one of the following expressions, state whether it is meaningful, and if it is, state whether it is a scalar field (i.e. scalar function) or a vector field. (you don't need to show work)

1. curl $f$
2. div $\mathbf{F}$
3. grad $\mathbf{F}$
4. div(grad $f$)
5. curl(curl $\mathbf{F}$)
6. (grad $f$) $\times$ (div $\mathbf{F}$)
7. grad $f$
8. curl(grad $f$)
9. grad(div $\mathbf{F}$)
10. div(div $\mathbf{F}$)
11. div(curl(grad $f$))

**13:** Determine whether or not the vector field $\mathbf{F}(x, y, z) = 5y^2z^3\mathbf{i} + 10xyz^3\mathbf{j} + 15xy^2z^2\mathbf{k}$, and, if it is, find a scalar function $f$ such that $\mathbf{F} = \nabla f$.