Homework Set 6

Due: Friday August 12

This assignment contains a lot of problems of parametrizing surfaces. Once you've found a parametrization, you can use Wolfram Mathematica or other similar software to plot your equations and see if they give you the correct picture.

Turn in the following problems:

16.6

19: Find a parametrization for the plane through the origin that contains the vectors $\mathbf{i} - \mathbf{k}$ and $\mathbf{j} - \mathbf{k}$.

24: Find a parametrization for the part of the sphere $x^2 + y^2 + z^2 = 9$ that lies between the planes $z = -2$ and $z = 2$.

25: Find a parametric representation for the part of the cylinder $y^2 + z^2 = 121$ that lies between the planes $x = 0$ and $x = 3$.

29: Find parametric equations for the surface obtained by rotating the curve $y = e^{-x}$, $0 \leq x \leq 3$ around the $x-$axis.

37: Find an equation of the tangent plane to the given parametric surface $\mathbf{r}(u, v) = u^2\mathbf{i} + 6u \sin v\mathbf{j} + u \cos v\mathbf{k}$ at the point $u = 2$, $v = 0$.

41: Find the area of the part of the plane $x + 2y + 3z = 1$ that lies inside the cylinder $x^2 + y^2 = 6$.(answer: $2\sqrt{14}\pi$)

45: Find the area of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 9$. (answer: $\frac{2}{3}(10\sqrt{10} - 1)\pi$)

16.7:

17: Evaluate the surface integral $\iint_S (x^2z + y^2z)dS$, where S is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$. (Answer: 16π).

Exercise: Evaluate the surface integral $\iint_S xy dS$, where S is the triangular region with vertices (1,0,0), (0,6,0), (0,0,6). (answer: $3\sqrt{19/2}$)

Exercise II: Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for the given vector fields $\mathbf{F}$ and the oriented surface S. In other words, find the flux of $\mathbf{F}$ across S. For closed surfaces use the positive (outward) orientation.

1. $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z^4\mathbf{k}$, S is the part of the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 1$ with downward orientation. (answer: $\pi/3$)
2. $\mathbf{F}(x, y, z) = x\mathbf{i} - z\mathbf{j} + y\mathbf{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 36$ in the first octant, with orientation toward the origin. (answer: -36π)
3. $\mathbf{F}(x, y, z) = xz\mathbf{i} + x\mathbf{j} + y\mathbf{k}$, S is the hemisphere $x^2 + y^2 + z^2 = 9$, $y \geq 0$, oriented in the direction of the positive y-axis. (answer: 0)
4. $\mathbf{F}(x, y, z) = x\mathbf{i} + 2y\mathbf{j} + 3z\mathbf{k}$, S is the cube with vertices $(\pm 1, \pm 1, \pm 1)$. (answer: 48)
5. $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + 9\mathbf{k}$, where S is the boundary of the region enclosed by the cylinder $x^2 + z^2 = 1$ and the planes $y = 0$ and $x + y = 8$ (answer: 16π).