Syllabus for MA 59500L
Lie Algebras
Spring 2026

1. Course Description

This is a basic introductory course on Lie algebras. Major topics to be covered:

- Manifolds and Lie groups
- Lie group actions and homogeneous spaces
- Vector fields, the exponential map, and three fundamental theorems of Lie theory
- The universal enveloping algebra and the Poincaré-Birkhoff-Witt theorem
- Free Lie algebras and the Baker-Campbell-Hausdorff formula
- Nilpotent and solvable Lie algebras, Engel's and Lie's theorems
- Simple, semisimple, reductive Lie algebras and their properties
- The Killing form and Cartan's criteria for solvability and semisimplicity
- The Casimir element, extensions of representations, and Whitehead's theorem
- Cartan subalgebras and the root space decomposition of semisimple Lie algebras
- Strongly regular elements and Chevalley's theorem
- Root systems and properties of their Weyl groups
- Classification of reduced root systems via Dynkin diagrams
- Classification of semisimple Lie algebras and their Chevalley-Serre presentation
- Representation theory of semisimple Lie algebras
- Weyl character and dimension formulas

If time remains: Levi's theorem, Harish-Chandra isomorphism, and BGG resolution.

Prerequisites: Basic notions from algebra, especially linear algebra (general familiarity with topology and manifold theory will be useful for the first 3 weeks).

2. Learning outcomes

Students will master the theory of Lie algebras, which is foundational for both mathematics and physics. We will start from Lie groups that provide an actual motivation for the subject, but will quickly diverge to the study of finite-dimensional Lie algebras, with the key emphasis placed on the semisimple Lie algebras (which admit an elegant complete theory). Upon completion of the course, students will be prepared to take advanced courses at graduate level.

3. Course Information

Time: MWF 2:30–3:20pm
Location: PHYS 110
Course CRN: #38376
Course Credits: 3
Course Webpage: here

4. Instructor Information

Instructor: Sasha Tsymbaliuk
Email: otsymbal@purdue.edu (emails will be responded within 24h Mon-Fri)
Office hours: Wed 12–1pm, Fri 3:30–4:30pm (Math 620) AND 1 flexible hour upon request

5. Grader Information

Grader: Kyungtak Hong
Email: hong420@purdue.edu
Office: MATH 739

6. Textbooks

There is no required textbook. The recommended references are:

(1) Book "An Introduction to Lie Groups and Lie Algebras" by Alexander Kirillov, Jr. Cambridge Studies in Advanced Mathematics (2017), ISBN:978-1316614105.
(2) Book "Introduction to Lie Algebras and Representation Theory" by James Humphreys Springer (1978), ISBN:0-387-90053-5.

Some other relevant literature is:

(3) Book "Introduction to Lie Algebras" by Karin Erdmann and Mark Wildon Springer Undergraduate Mathematics Series (2006), ISBN:978-1-84628-040-5.
(4) Book "Lie algebras and Lie groups" by Jean-Pierre Serre Lecture Notes in Mathematics (2006), ISBN:978-3-540-55008-2.
(5) Book "Lie algebras of finite and affine type" by Roger Carter Cambridge University Press (2005), ISBN:978-0-521-85138-1.
(6) Book "Representation Theory. A first course" by William Fulton and Joe Harris Springer (1999), ISBN:0-387-97527-6.

7. Assignments, Exams, and Grading Scale

There will be weekly homework assignments typically due at 11:59pm on Fridays, to be submitted online at Brightspace. The assignments will generally consist of 6–10 problems of various difficulty. There are no exams in this course.

For undergraduate students, there is an additional requirement to turn in a short (3-6 pages) expositional paper on one of the topics covered in the class (or a closely connected one). The choice of a topic shall be discussed with each student individually later in the term, and the final paper shall be due at the beginning of the Final Exam's week.

8. Grade Cutoffs

To get $A-$, a graduate student is expected to solve most of the simpler problems, while to get grades $A, A+$ one should also solve some of the harder/more technical problems (marked by an asterisk *). The University's policy is that students who get at least 97% of the total points in this course are guaranteed an A+, 93% guarantees an A, 90% an A-, 87% a B+, 83% a B, 80% a B-, 77% a C+, 73% a C, 70% a C-, 67% a D+, 63% a D, and 60% a D-. For each of these grades, the lower percentage will most probably suffice (as noted above).

9. Resources and Collaboration

While solutions are often available online, please make every effort to solve problems yourselves (in case you had to look up for hints or solutions, please cite the source accordingly).

You are welcome to work together on homework problems, but you should work separately when you write them up. Working hard and independently on the homework is the best way to absorb the material and get the most out of the course. Taking shortcuts on the homework is certain to harm your performance in the course.

You can use AI to explore mathematics, but you should not use it to do your homework.

10. Attendance

This course follows Purdue's academic regulations regarding attendance, which states that students are expected to be present for every meeting of the classes in which they are enrolled. When conflicts or absences can be anticipated, such as for many University-sponsored activities and religious observations, the student should inform the instructor of the situation as far in advance as possible. Purdue expects each student to be responsible for class-related work missed due to an unavoidable absence. Students should contact their instructors directly to discuss the absence and opportunity to complete missed coursework.

Please do not come to class if you are feeling ill, but do email the instructor with the subject line "Absence" (notify you are feeling sick and cannot come, no need to describe symptoms).

11. Drop and Add calendars

In accordance with the university policy, the course drop deadlines have been set at the end of week 13 (April 16 for Spring 2026). For other important dates, you can access the Drop/Add calendars by visiting:

`https://purdue.edu/registrar/calendars`

and scrolling down to locate the Drop/Add Refund & Deadline Calendars, or by following this direct link:

`https://catalog.purdue.edu/preview_program.php?catoid=18&poid=33634`

12. Academic Adjustments for Students with Disabilities

Purdue University strives to make learning experiences accessible to all participants. If you anticipate or experience physical or academic barriers based on disability, you are encouraged to contact the Disability Resource Center at: drc@purdue.edu or by phone: 765-494-1247, as soon as possible.

If the Disability Resource Center (DRC) has determined reasonable accommodations that you would like to utilize in this class, you must send your Course Accommodation Letter to the instructor. Instructions on sharing your Course Accommodation Letter can be found by visiting: `https://www.purdue.edu/drc/students/course-accommodation-letter.php`. Additionally, you are strongly encouraged to contact the instructor as soon as possible to discuss implementation of your accommodations.

13. Non-discrimination

Purdue University is committed to maintaining a community which recognizes and values the inherent worth and dignity of every person; fosters tolerance, sensitivity, understanding, and mutual respect among its members; and encourages each individual to strive to reach his or her own potential. In pursuit of its goal of academic excellence, the University seeks to develop and nurture diversity. The University believes that diversity among its many members strengthens the institution, stimulates creativity, promotes the exchange of ideas, and enriches campus life.

14. Mental Health/Wellness Statement

Purdue University is committed to advancing the mental health and well-being of its students. If you or someone you know is feeling overwhelmed, depressed, and/or in need of mental health support, services are available. For help, such individuals should contact Counseling and Psychological Services (CAPS) at 765-494-6995 during and after hours, on weekends and holidays, or by going to the CAPS office on the second floor of the Purdue University Student Health Center (PUSH) during business hours.

15. Academic integrity

Because students have taken the official Purdue Honor Pledge, I expect there to be no academic dishonesty in this class. Students will not be asked to sign or repeat any similar statement again and I will treat you with the trust deserving of someone who has taken the pledge. However, I am required to mention in my syllabus material that Purdue policy states: *"Incidents of academic misconduct in this course will be addressed by the course instructor and referred to the Office of Student Rights and Responsibilities (OSRR) for review at the university level. Any violation of course policies as it relates to academic integrity will result minimally in a failing or zero grade for that particular assignment, and at the instructor's discretion may result in a failing grade for the course. In addition, all incidents of academic misconduct will be forwarded to OSRR, where university penalties, including removal from the university, may be considered"*

16. Emergency Preparation

In the event of a major campus emergency, course requirements, deadlines and grading percentages are subject to changes that may be necessitated by a revised semester calendar or other circumstances beyond the instructor's control. Relevant changes to this course will be posted onto the course website or can be obtained by contacting the instructor via email. an active threat such as a shooting.