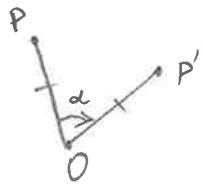


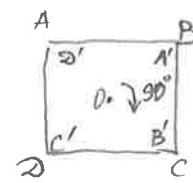
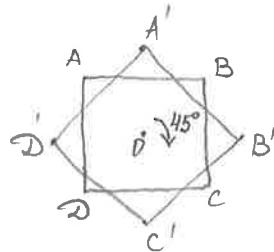
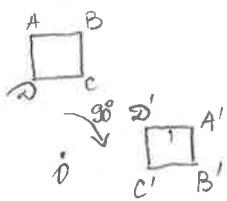
- 2<sup>nd</sup> type of rigid motions in 2d: "Rotations"

A rotation in the plane is a rigid motion that pivots/swings an object around a fixed point  $O$ . It is defined by 2 pieces of information.

- (1) point  $O$  - the rotocenter, (2) angle of rotation.

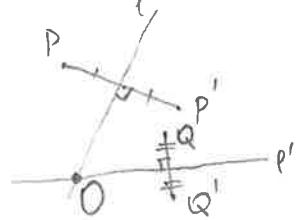


### Example:



### Key properties of rotations

- The rotocenter is the only fixed point of any rotation, which is not an identity map
- The rotation is completely determined by  $O$  and  $\alpha$
- The rotation is completely determined by any two points  $P, Q$  and their images  $P', Q'$  (as far as  $P \neq P'$ ,  $Q \neq Q'$ )



The rotocenter  $O$  is obtained as intersection of two lines  $l$  and  $l'$  which pass through middle points of  $PP'$ ,  $QQ'$  and are perpendicular to them.

The angle of rotation  $\alpha = \angle POP'$ .

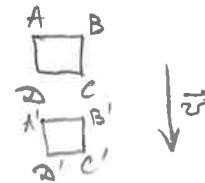
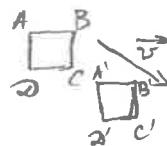
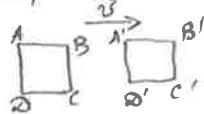
- A  $360^\circ$  rotation is the identity motion  $\rightarrow$  rotation by  $\alpha = n \cdot 360^\circ + \beta$  ( $n$ -integer) is equivalent to a rotation by  $\beta$  with the same rotocenter
- A rotation is a proper rigid motion, i.e. preserves orientation

3<sup>rd</sup> type of rigid motions : "Translations"

A translation consists of essentially dragging an object in a specified direction and by a specified amount. These two pieces of information are combined in the form of a vector of translation.

A vector is represented by an arrow pointing in the direction of translation, while its length is the length of translation.

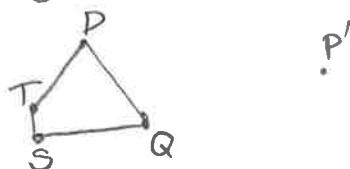
Example



Key properties of translations

- A translation is completely determined by any point  $P$  and its image  $P'$ .  
 $\vec{v} = \overrightarrow{PP'}$
- A translation (which is not an identity map) has no fixed points.
- A translation is a proper rigid motion, i.e. it preserves an orientation.
- A translation by  $\vec{v}$  followed by a translation in the opposite direction by  $-\vec{v}$  is the identity map.

Ex: Find an image of the shape F under a translation given  $P$  and  $P'$ :



! Discuss several more examples!