Laplace Transform

Suppose that the function $f(t)$ is continuous and piecewise smooth for $t \geq 0$ and is of exponential order as $t \to \infty$: there exist nonnegative constants M, c and T such that

$$|f(t)| \leq Me^{ct}, \quad t \geq T.$$

Then there exists the Laplace transform $F(s) = L(f(t))(s) = \int_0^\infty e^{-st} f(t)dt$.

A Table of Laplace Transform

(1) $L(t^n) = \frac{n!}{s^{n+1}}, \quad s > 0$ (n: a nonnegative integer)

(2) $L(t^a) = \frac{\Gamma(a+1)}{s^{a+1}}, \quad s > 0 \quad (a > -1)$

(3) $L(e^{\alpha t}) = \frac{1}{s-\alpha}, \quad s > \alpha$

(4) $L(\cos(bt)) = \frac{s}{s^2+b^2}, \quad s > 0$

(5) $L(\sin(bt)) = \frac{b}{s^2+b^2}, \quad s > 0$

(6) $L(\cosh(kt)) = \frac{s}{s^2-k^2}, \quad s > |k|$

(7) $L(\sinh(kt)) = \frac{k}{s^2-k^2}, \quad s > |k|$

Formulas

(1) $\Gamma(x) = \displaystyle\int_0^\infty e^{-t} t^{x-1} dt, \qquad \Gamma(x+1) = x\Gamma(x)$

(2) $L(f^{'}(t)) = sL(f) - f(0) \quad \text{for} \quad s > c$

$L(f''(t)) = s^2 L(f) - sf(0) - f'(0) \quad \text{for} \quad s > c$

(3) $L(u_c(t)f(t-c)) = e^{-cs}F(s), \qquad L(u_c(t)) = \dfrac{e^{-cs}}{s}$

(4) $L(e^{at}f(t)) = F(s-a) \quad \text{for} \quad s > a + c$