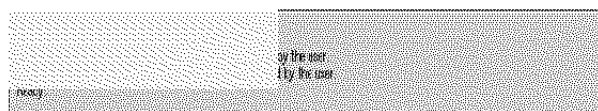
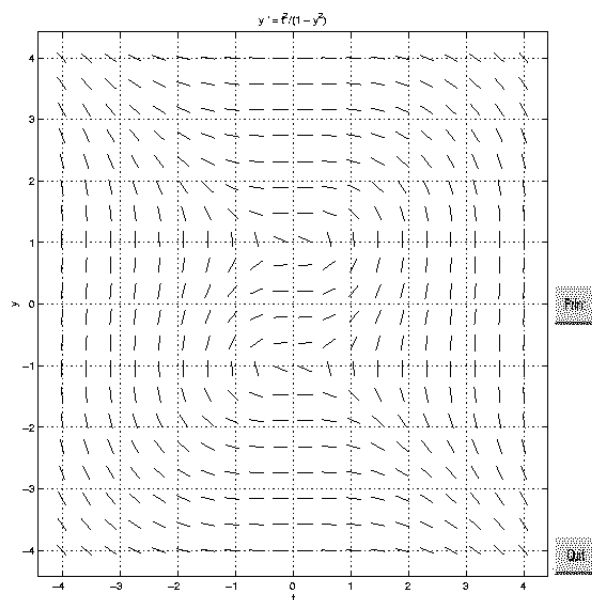


MA266 Practice Problems

- If $y' + (1 + \frac{1}{t})y = \frac{1}{t}$ and $y(1) = 0$, then $y(\ln 2) =$
A. $\ln 2 - \ln(\ln 2)$ **B.** $\ln(\ln 2)$ **C.** $\ln(\ln 2) + \frac{1}{2\ln 2}$ **D.** $\frac{1}{\ln 2} \left(1 - \frac{e}{2}\right)$ **E.** $\frac{1}{\ln 2} - 1$
- What is the largest open interval for which a unique solution of the initial value problem $ty' + \frac{1}{t+1}y = \frac{t-2}{t-3}$, $y(1) = 0$ is guaranteed?
A. $0 < t < 1$ **B.** $0 < t < 2$ **C.** $0 < t < 3$ **D.** $-1 < t < 3$ **E.** $-1 < t < 1$
- Use the dfield plot below to estimate where the solution of $y' = \frac{t^2}{1-y^2}$, $y(0) = 0$ is defined:



- A.** $-1.2 < t < 1.2$ **B.** $-4 < t < 4$ **C.** $-1 < t < 2$ **D.** $-2 < t < 2$ **E.** $-4 < t < \infty$
- Consider the autonomous differential equation $\frac{dy}{dt} = -\frac{1}{10}(y-1)(y-4)^2$. Classify the stability of each equilibrium solution.
A. $y = 1$ and $y = 4$ both unstable **B.** $y = 4$ stable; $y = 1$ unstable **C.** $y = 0$ and $y = 1$ stable; $y = 4$ unstable **D.** $y = 1$ stable; $y = 4$ unstable **E.** $y = 0$ stable; $y = 1$ and $y = 4$ unstable
- Determine whether $x + 2y + (2x + y)\frac{dy}{dx} = 0$ is separable, homogeneous, linear and/or exact.
A. LINEAR and SEP **B.** SEP and HOM **C.** HOM and EXACT **D.** LINEAR and HOM **E.** LINEAR, HOM and EXACT

6. An explicit solution of $y' = y^2 - 1$ is
A. $y = \frac{Ce^{2t}}{1 - Ce^{2t}}$ **B.** $y = \frac{1 + Ce^{2t}}{1 - Ce^{2t}}$ **C.** $y = \frac{1}{1 - Ce^{2t}}$ **D.** $y = \frac{1 + Ce^{2t}}{1 - e^{2t}}$ **E.** $\frac{y^3}{3} - y = C$
7. If $y' = y^3$ and $y(0) = 1$, then $y(-1) =$
A. $5^{-\frac{1}{4}}$ **B.** $\frac{1}{\sqrt{3}}$ **C.** $\sqrt{3}$ **D.** 1 **E.** Does not exist
8. If $(2x^2 + y^2)dx - xy dy = 0$ and $y(1) = 2$, then $y(e^3) =$
A. $2e^9$ **B.** $e^3\sqrt{10}$ **C.** $2e^3$ **D.** $4e^3$ **E.** $16e^3$
9. An implicit solution of $y^2 + 1 + (2xy + 1)\frac{dy}{dx} = 0$ is
A. $2(xy^2 + y) = C$ **B.** $xy^2 + y = C$ **C.** $xy^2 + x + y = C$ **D.** $\frac{y^3}{3} + y + x^2y + x = C$ **E.** $y = xy^2 + C$
10. If y' is proportional to y , $y(0) = 2$ and $y(1) = 8$. For what value of t does $y(t) = 20$?
A. $\ln 6$ **B.** $\ln 4$ **C.** $\frac{\ln 8}{\ln 2}$ **D.** $\ln \frac{5}{2}$ **E.** $\frac{\ln 10}{\ln 4}$
11. The general solution of $y'' - 4y' + 4y = 0$ is
A. $y = C_1e^{2t} + C_2te^{2t}$ **B.** $y = C_1e^{2t} + C_2e^{2t}$ **C.** $y = C_1e^{2t} + C_2e^{-2t}$ **D.** $y = C_1e^{-2t} + C_2te^{-2t}$ **E.** $y = C_1t + c_2t^2$
12. The general solution of $y''' + 4y'' + 5y' = 0$ is
A. $y = C_1e^{-2t}\cos t + C_2e^{-2t}\sin t$ **B.** $y = C_1 + C_2e^{-2t}\cos t + C_3e^{-2t}\sin t$ **C.** $y = C_1 + C_2e^t\cos 2t + C_3e^t\sin 2t$ **D.** $y = C_1 + C_2\cos t + C_3\sin t$ **E.** $y = C_1 + C_2e^{2t}\cos t + C_3e^{2t}\sin t$
13. A particular solution, y_p , of $y'' - 4y' + 3y = 2t + e^t$ is
A. $\frac{2}{3}t + \frac{8}{9} - \frac{1}{2}te^t$ **B.** $\frac{2}{3}t + \frac{1}{2} - \frac{1}{2}te^t$ **C.** $\frac{1}{3}t + \frac{1}{2} - \frac{1}{2}te^t$ **D.** $\frac{1}{3}t + \frac{1}{2} - \frac{1}{2}e^t$ **E.** $t^2 + e^t$
14. If $y'' + 5y' + 6y = 24e^t$, $y(0) = 0$, $y'(0) = 0$, then $y(1) =$
A. $-e^{-2} + 6e^{-3} + e$ **B.** $-8e^{-2} + 6e^{-3} + e$ **C.** $8e^{-2} + e^{-3} + e$ **D.** $-8e^{-2} + 6e^{-3} + 2e$ **E.** 0
15. The differential equation $y'' - \frac{2}{t}y' + \frac{2}{t^2}y = 0$ has solutions $y_1(t) = t$ and $y_2(t) = t^2$. If $y'' - \frac{2}{t}y' + \frac{2}{t^2}y = 2$, $y(1) = 0$ and $y'(1) = 0$, then $y(2) =$
A. 0 **B.** -6 **C.** $8\ln 2$ **D.** $8\ln 2 - 4$ **E.** $8\ln 2 + 4$
16. An object weighing 8 pounds attached to a spring will stretch it 6 inches beyond its natural length. There is a damping force with a damping constant $c = 6$ lbs-sec/ft and there is no external force. If at $t = 0$ the object is pulled 2 feet below equilibrium and then released, the initial value problem describing the vertical displacement $x(t)$ becomes :

$$\begin{array}{lll}
\text{A. } \begin{cases} 8x'' + 6x' + 16x = 0 \\ x(0) = 2 \\ x'(0) = 0 \end{cases} & \text{B. } \begin{cases} 8x'' + 6x' + 16x = 0 \\ x(0) = -2 \\ x'(0) = 0 \end{cases} & \text{C. } \begin{cases} \frac{1}{4}x'' + 6x' + 16x = 0 \\ x(0) = 2 \\ x'(0) = 0 \end{cases} \\
\text{D. } \begin{cases} \frac{1}{4}x'' + 6x' + 8x = 0 \\ x(0) = 2 \\ x'(0) = 0 \end{cases} & \text{E. } \begin{cases} 256x'' + 6x' + 16x = 0 \\ x(0) = 2 \\ x'(0) = 0 \end{cases} &
\end{array}$$

17. A spring-mass system is governed by the initial value problem $x'' + 4x' + 4x = 4 \cos \omega t$, $x(0) = 0$, $x'(0) = -2$. For what value(s) of ω will resonance occur ?

A. 0 B. 2 C. 4 D. $2 < \omega < \infty$ E. no value of ω

18. A tank initially contains 40 ounces of salt mixed in 100 gallons of water. A solution containing 4 oz of salt per gallon is then pumped into the tank at a rate of 5 gal/min. The stirred mixture flows out of the tank at the same rate. How much salt is in the tank after 20 minutes ?

A. 20 B. 80 C. $40 + 20e$ D. $400 - 360e^{-1}$ E. $400 + 360e^2$

19. Rewrite the second order equation $2u'' + 3u' + ku = \cos 2t$ as a system of 1st order equations.

$$\begin{array}{ll}
\text{A. } \begin{cases} x' = y \\ y' = \frac{1}{2}(-3y - kx + \cos 2t) \end{cases} & \text{B. } \begin{cases} x' = y \\ y' = \frac{1}{2}(-3x - ky + \cos 2t) \end{cases} \\
\text{C. } \begin{cases} x' = x \\ y' = \frac{1}{2}(-3y - kx + \cos 2t) \end{cases} & \text{D. } \begin{cases} x' = y \\ y' = 2y + kx + \cos 2t \end{cases} \\
\text{E. } \begin{cases} x' = 2y + 3x + \cos 2t \\ y' = x \end{cases} &
\end{array}$$

20. The solution of $X' = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} X$, $X(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ is

$$\begin{array}{lll}
\text{A. } 2e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} & \text{B. } 2e^{3t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + e^{-t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} & \text{C. } e^{3t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\
\text{D. } 3e^{3t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} - e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} & \text{E. } 3e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + e^{-t} \begin{pmatrix} 0 \\ -4 \end{pmatrix} &
\end{array}$$

21. Solve $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $X(0) = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$.

$$\begin{array}{ll}
\text{A. } X(t) = 2e^t \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} - e^t \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} & \text{B. } X(t) = 2e^t \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} + e^t \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \\
\text{C. } X(t) = 2e^t \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} - e^t \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} & \text{D. } X(t) = e^t \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} - e^t \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \\
\text{E. } X(t) = e^t \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix} - e^t \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} &
\end{array}$$

22. Solve the initial value problem $\vec{x}'(t) = A\vec{x}(t)$, $\vec{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ where $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

$$\begin{array}{lll}
\text{A. } e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix} - 2te^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} & \text{B. } e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 2te^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} & \text{C. } e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + te^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
\text{D. } e^t \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 2te^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} & \text{E. } e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix} - 2te^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} &
\end{array}$$

23. Find a particular solution of $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

A. $X_p = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ **B.** $X_p = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ **C.** $X_p = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ **D.** $X_p = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$
E. $X_p = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

24. Find the general solution of $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} 6e^{-t} \\ 1 \end{pmatrix}$.

A. $C_1 e^t \begin{pmatrix} 0 \\ 1 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
B. $C_1 e^t \begin{pmatrix} 0 \\ 1 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 6e^{-t} \\ 1 \end{pmatrix}$
C. $C_1 e^t \begin{pmatrix} 0 \\ 1 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
D. $C_1 e^t \begin{pmatrix} 0 \\ 1 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
E. $C_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

25. $\mathcal{L}\{e^t(1 + \cos 2t)\} =$

A. $\frac{1}{s-1} + \frac{1}{(s-1)^2 + 4}$ **B.** $\frac{1}{s-1} + \frac{s-1}{s^2 - 2s + 5}$ **C.** $\left(\frac{1}{s-1}\right) \left(\frac{1}{s} + \frac{s-1}{(s-1)^2 + 4}\right)$
D. $\left(\frac{1}{s-1}\right) \left(\frac{s-1}{s^2 - 2s + 5}\right)$ **E.** $\frac{1}{s} + \frac{s}{(s-1)^2 + 4}$

26. Find the Laplace transform of $f(t) = \begin{cases} t, & 0 \leq t < 1 \\ 0, & 1 \leq t < \infty \end{cases}$.

A. $e^{-s} \left(\frac{1}{s} + \frac{1}{s-2}\right)$ **B.** $\frac{1}{s^2} + 2e^{-s} \left(\frac{1}{s} + \frac{1}{s^2}\right)$ **C.** $\frac{1}{s^2} - e^{-s} \frac{1}{s^2}$ **D.** $\frac{1}{s^2} - e^{-s} \left(\frac{1}{s} + \frac{1}{s^2}\right)$
E. $e^{-s} \left(\frac{1}{s} + \frac{1}{s^2}\right)$

27. Solve $y'' + 3y' + 2y = 4u_1(t)$, $y(0) = 0$, $y'(0) = 1$

A. $u_1(t) \left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right)$ **B.** $u_1(t) \left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right) + e^{-t} - e^{-2t}$
C. $u_0(t) \left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right) + e^{-t} - e^{-2t}$ **D.** $\left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right) + e^{-t} - e^{-2t}$
E. $e^{-t} - e^{-2t}$

28. Find the solution of the initial value problem $y'' + y = \delta(t - \pi)$, $y(0) = 0$, $y'(0) = 1$.

A. $y = \sin t + u_0(t) \sin t$ **B.** $y = \sin t + u_\pi(t) \sin \pi t$ **C.** $y = \sin t + u_\pi(t) \sin(t - \pi)$
D. $y = u_\pi(t)(\sin t + \sin(t - \pi))$ **E.** $y = u_\pi(t) \sin t$

29. The inverse Laplace transform of $F(s) = \frac{se^{-s}}{s^2 + 2s + 5}$ is
- A. $u_1(t) (e^{-t} \cos 2(t-1)) - \frac{1}{2} e^{-t} \sin 2(t-1)$ B. $(e^{-t+1} \cos 2(t-1)) - \frac{1}{2} e^{-t+1} \sin 2(t-1)$
 C. $u_1(t) (e^{t-1} \cos 2(t-1)) - \frac{1}{2} e^{t-1} \sin 2(t-1)$ D. $u_0(t) (e^{-t} \cos 2t) - \frac{1}{2} e^{-t} \sin 2t$
 E. $u_1(t) (e^{-t+1} \cos 2(t-1) - \frac{1}{2} e^{-t+1} \sin 2(t-1))$

30. $\mathcal{L} \left\{ \int_0^t \sin 2(t-\tau) \cos(3\tau) d\tau \right\} =$

- A. $\frac{2s}{(s^2+4)(s^2+9)}$ B. $\frac{2}{s^2+4} + \frac{s}{s^2+9}$ C. $\frac{1}{s^2+4} + \frac{s}{s^2+9}$ D. $\frac{2}{(s^2+4)(s^2+9)}$
 E. $\frac{s}{(s^2+4)(s^2+9)}$

31. The phase portrait of the system $\vec{x}'(t) = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \vec{x}(t)$, whose general solution is

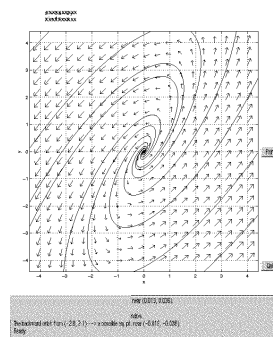
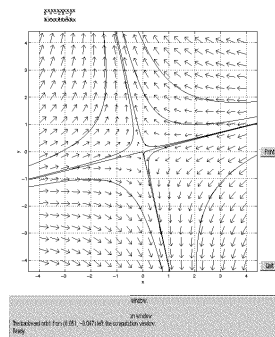
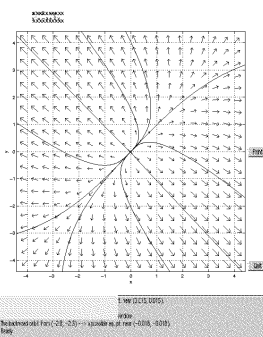
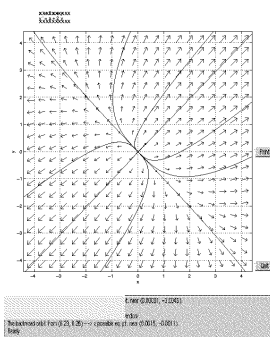
$$\vec{x}(t) = C_1 e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \text{ looks most like :}$$

A.

B.

C.

D.



Answers

- | | | |
|-------|-------|-------|
| 1. D | 11. A | 21. C |
| 2. C | 12. B | 22. C |
| 3. A | 13. A | 23. A |
| 4. D | 14. D | 24. D |
| 5. C | 15. D | 25. B |
| 6. B | 16. C | 26. D |
| 7. B | 17. E | 27. B |
| 8. D | 18. D | 28. C |
| 9. C | 19. A | 29. E |
| 10. E | 20. A | 30. A |
| | | 31. B |

$f(t) = \mathcal{L}^{-1}\{F(s)\}$		$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$
2.	e^{at}	$\frac{1}{s-a}$
3.	t^n	$\frac{n!}{s^{n+1}}$
4.	$t^p \ (p > -1)$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin at$	$\frac{a}{s^2 + a^2}$
6.	$\cos at$	$\frac{s}{s^2 + a^2}$
7.	$\sinh at$	$\frac{a}{s^2 - a^2}$
8.	$\cosh at$	$\frac{s}{s^2 - a^2}$
9.	$e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}$
10.	$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}$
11.	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\frac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right) \ c > 0$
16.	$\int_0^t f(t-\tau)g(\tau) d\tau$	$F(s)G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$