MA 266 Practice Test 1

Name .....

ID number .....

INSTRUCTIONS in the Test

1. **Do not open this exam booklet until told to do so**.
2. **There are 7 or 8 problems**
3. **Show all your work - if you need more space, continue on the back of the page for that problem. No work no credit.**
4. **Show your final answer by enclosing it in a box or circle**.

Problem 1. (8 points) Find the general solution of the ODE

$$y' + 4y = e^{-2t}.$$

Justify all your answers. Box your answers.

ANSWER:

$$y(t) = \frac{1}{2}e^{-2t} + Ce^{-4t}.$$

Problem 2. (8 points) Find the general solution of the ODE

$$ty' - 2y = -2t^3e^{2t}.$$

Justify all your answers. Box your answers.

ANSWER:

$$y(t) = -t^2e^{2t} + Ct^2.$$

Problem 3. (8 points) Find the general solution of

$$\frac{dy}{dx} = \frac{xy + 2x + y + 2}{(y+2)^2},$$

Justify all your answers. Box your answers.

ANSWER:

$$y(t) = -2 \pm \sqrt{x^2 + 2x + C}.$$

Problem 4. (8 points) Find the general solution of

$$\frac{dy}{dx} = \frac{3y - 2x}{y}.$$

Justify all your answers. Box your answers.

ANSWER:

$$y(t) = \frac{1}{2}\left(4x + C \pm \sqrt{4Cx + C^2}\right).$$

Problem 5. (8 points) A tank contains 200 gal (gallons) of liquid. Initially, the tank contains pure water. At time $t = 0$, brine containing 3 lb/gal salt begins to pour into the tank at a rate of 2 gal/min, and the well-stirred mixture is allowed to drain away at the same rate. How many minutes must elapse before there are 100 lb of salt in the talk?

A. 600 B. $600 - 1/e$ C. $600 - 600/e$ D. $100\ln(6/5)$ E. $-100\ln(400)$

ANSWER: D

Problem 6. (10 points) What is the largest open interval in which a solution to the initial value problem

$$(t-1)y' + \sqrt{t+2}y = \frac{3}{t-3}, \qquad y(2) = -5$$

is guaranteed to exist?

A. $(1, 3)$ B. $(2, 3)$ C. $(-2, 3)$ D. $(-2, 1)$ E. $(-2, \infty)$

ANSWER: A

Problem 7. (10 points) Find all the asymptotically stable equilibrium solutions for the autonomous differential equation

$$y' = (y^2 - 1)(4 - y^2).$$

ANSWER: $y = -1$ and $y = 2$.

Problem 8. (10 points) A tank with capacity 500 gal originally contains 100 gallons of water with a salt concentration of $1/2$ lb/gal. A solution containing a salt concentration of 2 lb/gal enters at a rate of 2 gal/min and the well-stirred mixture is pumped out at the rate of 1 gal/min. What is the amount of salt in the tank after 50 min.

ANSWER: $Q(50) = 200(lb)$.

Problem 9. (10 points) A skydiver weighing 200 lb (with mass $25/4$ lb) falls vertically downward from an altitude of 4000 ft and opens the parachute after 10 seconds of free fall. Assume that the force of air resistance, which is directed opposite to the velocity, is 0.8 $|v|$ when the parachute is closed and $12|v|$ when the parachute is open, where the velocity v is measured in ft/sec. Use $g = 32ft/sec^2$.

(a) Find the speed of the skydiver when the parachute openes.

(b) Find the distance fallen before the parachute opens.

ANSWER: (1) $v(10) = 250(1 - e^{-1.28})$ (2) ≈ 1089.9.

Problem 10. (10 points) Find the implicit solution to the initial value problem

$$(e^x \sin(y) - 2y\sin(x) - 1) + (e^x \cos(y) + 2\cos(x) + 3)y' = 0, \qquad y(0) = \pi.$$

ANSWER:

$$e^x \sin(y) + 2y\cos(x) - x + 3y = C.$$

Problem 11. (10 points) What is the general solution to this differential equation?

$$\frac{dy}{dx} = (x - y)^2 + 1.$$

(Hint: use the substitution $v(x) = x - y(x)$.)

ANSWER:

$$y(x) = x - \frac{1}{x - C}.$$

Problem 12. (10 points) Find the general solution of the differential equation

(a) $3y'' + 2y' - 8y = 0$ (b) $y'' + 6y' + 10y = 0$

ANSWER:

(1) $y(t) = C_1 e^{-2t} + C_2 e^{4t/3}$ (2) $y(t) = C_1 e^{-3t}\cos(t) + C_2 e^{-3t}\sin(t)$.

Problem 13. (10 points) Which of the following statements is true about every solution of $y'' = -4y$?

A. $\lim_{t\to\infty} y(t) = +\infty$

B. y is a periodic function

C. $\lim_{t\to\infty} y(t) = 0$

D. $-1 \le y(t) \le 1$

E. $\lim_{t\to 0} y(t) = 0$

ANSWER: B

Problem 14. (10 points) Compute the Wronskian of the two functions.

(1) $f(x) = e^{2x}$ and $g(x) = 3x$.

(2) $f(x) = e^{3x}\cos(x)$ and $g(x) = e^{3x}\sin(x)$.

ANSWER:

(1) $W(f,g) = (3-6x)e^{2x}$ (2) $W(f,g) = e^{6x}$.