MA 266 Practice Test 2

Name ....

ID number ....

INSTRUCTIONS in the Test

1. **Do not open this exam booklet until told to do so.**
2. **There are 7 or 8 problems**
3. **Show all your work - if you need more space, continue on the back of the page for that problem. No work no credit.**
4. **Show your final answer by enclosing it in a box.**

Problem 1. (8 points) Find a fundamental set of solutions to the homogeneous differential equation

$$y^{(4)} + 8y'' + 16y = 0.$$

Justify all your answers. Box your answers.

Answer: $\{\cos(2t), \sin(2t), t\cos(2t), t\sin(2t)\}$

Problem 2. (8 points) Find the general solution of

$$y^{(4)} - 16y = 0.$$

Justify all your answers. Box your answers.

Answer: $y(t) = C_1e^{2t} + C_2e^{-2t} + C_3\cos(2t) + C_4\sin(2t)$

Problem 3. (8 points) Find the general solution of the equation

$$t^2y'' - ty' + y = 0.$$

Justify all your answers. Box your answers.

Answer: $y(t) = C_1t + C_2t\ln(t)$

Problem 4. (8 points) Given that a general solution of the homogeneous equation $t^2y'' + 3ty' - 3y = 0$ on the domain $t > 0$ is $y = c_1t + c_2t^{-3}$, Find a particular solution of the second order equation

$$t^2y'' + 3ty' - 3y = 16t.$$

Justify all your answers. Box your answers.

Answer: $Y(t) = 4t\ln(t)$

Problem 5. (10 points) One particular solution of the differential equation

$$y'' - 2y' + 2y = \frac{1}{\sin(t)}, \qquad 0 < t < \pi$$

is

A. $1/\sin(t)$

B. $\cos(t) + e^t \sin(t) \int e^{-t} \cos(t) \sin(t) dt$

C. $\cos(t) + e^t \sin(t) \int e^t \cos(t) \sin(t) dt$

D. $\cos(t) + e^t \sin(t) \int \cos(t) \sin(t) dt$

E. $1/\cos(t) + 1/\sin(t)$

Justify all your answers. Box your answers.

Answer: B

Problem 6. (8 points) According to the method of undetermined coefficients, what is the proper form of a particular solution Y to the following differential equation?

$$y^{(4)} - y''' - y'' + y' = t^2 + 4 + te^t.$$

(Hint: $r^4 - r^3 - r^2 + r = r(r-1)^2(r+1)$.)

A. $At^2 + Bt + C + Dt^3e^t$

B. $At^2 + Bt + C + Dt^2e^t + Ete^t$

C. $At^3 + Bt^2 + Ct + Dt^2e^{-t} + Ete^{-t}$

D. $At^3 + Bt^2 + Ct + Dt^3e^t + Et^2e^t$

E. $At^3 + Bt^2 + Ct + Dt^3e^t$

Justify all your answers. Box your answers.

Answer: D

Problem 7. (10 points) A mass of $3kg$ hangs from a spring with spring constant of $12kg/sec^2$. Suppose you pull the mass down an additional $5cm$ from its equilibrium position, and then release it with an initial velocity of $2cm/sec$ upwards. Find the amplitude and period of the resulting oscillatory motion.

Justify all your answers. Box your answers.

Answer: The amplitude $= \sqrt{26}$ and the period $= \pi$ (sec)

Problem 8. (8 points) Find the Laplace transform of the function

$$f(t) = \begin{cases} \sin(t), & 0 \le t < \pi \\ \cos(t), & t \ge \pi \end{cases}$$

(Hint: Recall that $\sin(\pi - t) = \sin(t)$ and $\cos(\pi - t) = -\cos(t)$.)

A. $\dfrac{1 - e^{\pi s}s}{s^2+1}$ B. $\dfrac{1 - e^{-\pi s}(s-1)}{s^2+1}$ C. $\dfrac{1 + e^{-\pi s}(s-1)}{s^2+1}$ D. $\dfrac{1 + e^{-\pi s}s}{s^2+1}$

Justify all your answers. Box your answers.

Answer: B

Problem 9. (10 points) Find the Laplace transform of the function

$$f(t) = \int_0^t \cos(t-\tau)e^{\tau}\sin(\tau)d\tau.$$

Justify all your answers. Box your answers.

Answer:

$$\frac{s}{(s^2+1)((s-1)^2+1)}$$

Problem 10. (10 points) Find the inverse Laplace transform of

$$F(s) = e^{-2s}\frac{2s+1}{s^2-4s+5}.$$

Justify all your answers. Box your answers.

Answer: $L^{-1}(F) = u_2(t)e^{2(t-2)}\left(2\cos(t-2) + 5\sin(t-2)\right)$

Problem 11. (8 points) Let $y(t)$ be the solution to the initial value problem

$$y'' + y = u_1(t), \qquad y(0) = 0, \quad y'(0) = 0.$$

Then find $y(\pi+1)$.

Justify all your answers. Box your answers.

Answer: $y(\pi+1) = 2$

Problem 12. (10 points) Find the solution of the initial value problem

$$y'' - 3y' + 2y = \delta(t-2), \qquad y(0) = 0, \quad y(0) = 1.$$

Justify all your answers. Box your answers.

Answer: $y(t) = e^{2t} - e^{t} + u_2(t)(e^{2t-4} - e^{t-2})$