MA 16010 Lesson 1: Precalculus review

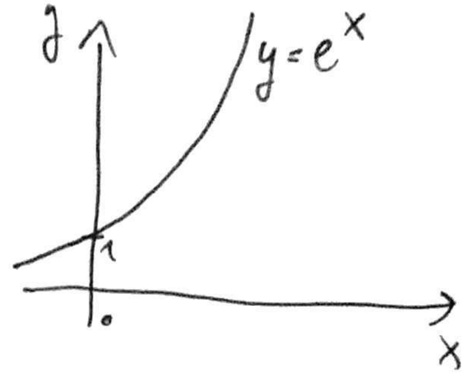
Exponentiation. For numbers a, b, we consider a^b either

when $a > 0$ or $a \leq 0$, b = rational number with odd ~~numerator~~ denominator

Examples:

$3^2, 1^{15}, \pi^3, 2^{\pi}, 5^{-\frac{1}{2}}, \ldots, (-1)^2, (-3)^{\frac{1}{3}}, \ldots$

Properties of exponentiation:



- $a^b \cdot a^c = a^{b+c}$
- $a^b / a^c = a^{b-c}$
- $a^{-b} = \frac{1}{a^b}$, $a^0 = 1$
- $(a^b)^c = a^{b \cdot c}$
- $a^{\frac{1}{b}} = \sqrt[b]{a}$

An *exponential function* is a function of the form $f(x) =$ a^x. The "best one" is the *natural exponential function* $f(x) =$ $e^x = \exp(x)$, where $e \approx 2.71828\ldots$ is the so-called Euler's number.

Exercise: Simplify the following expressions:

$e^5 e^{-3} = e^{(5+(-3))} = e^2$

$(e^{-2x})^5 = e^{(-2x)\cdot 5} = e^{-10x}$

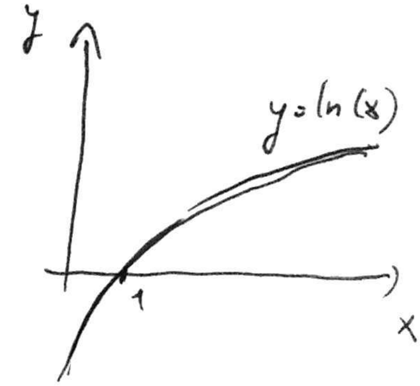
$\frac{e^{4x}e^3}{e^{7x}} = \frac{e^{4x+3}}{e^{7x}} = e^{4x+3-7x} = e^{3-3x}$

Logarithm. The function $f(x) = \ln(x)$ is defined as the inverse function for $g(x) = e^x$. It is called *the (natural) logarithm function.*

The domain of $\ln(x)$ is the set of all positive numbers $(\{x \mid x > 0\})$.

Properties of logarithm:

- $\ln(e^x) = x, \quad e^{\ln(x)} = x$
- $\ln(x \cdot y) = \ln(x) + \ln(y)$
- $\ln(x/y) = \ln(x) - \ln(y), \quad \ln(1/x) = -\ln(x)$
- $\ln(x^n) = n \cdot \ln(x)$
- $\ln(\sqrt[n]{x}) = \frac{1}{n} \cdot \ln(x)$



Exercise: Simplify the following expressions:

$\ln(3x) + \ln(5) = \ln(\cancel{15}(3x) \cdot 5) = \ln(15x) \; (= \ln(x) + \ln(15))$

$\ln(5x) - \frac{1}{3}\ln(y) = \ln(5x) - \ln(\sqrt[3]{y}) = \ln\left(\frac{5x}{\sqrt[3]{y}}\right)$

$\ln(e^{3x}) = 3x$

$e^{x\ln(5)} = \left(e^{\ln(5)}\right)^x = 5^x$

Exercise: Find all solutions to the equation: $\ln(2x^2) = 10$.

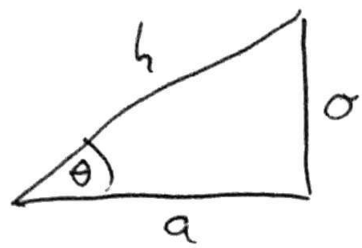
$\ln(2x^2) = 10$ take $\exp(-)$ on both sides

$2x^2 = e^{10}$

$x^2 = e^{10}/2$

$x = \pm\sqrt{\frac{e^{10}}{2}} \rightsquigarrow \underline{\underline{x = \pm \frac{e^5}{\sqrt{2}}}}$

Trigonometric functions.

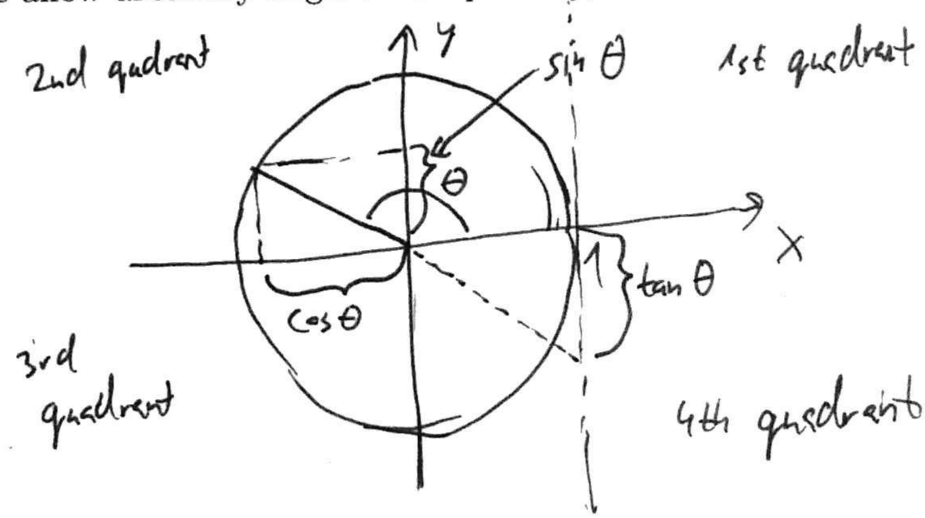


Given a right triange with an angle θ, adjacent side of length a, opposite side of length o and hypotenuse of length h, we have

$$\sin(\theta) = \underline{\ \frac{o}{h}\ }, \quad \cos(\theta) = \underline{\ \frac{a}{h}\ }, \quad \tan(\theta) = \underline{\ \frac{o}{a}\ },$$

(*) $$\sec(\theta) = \underline{\ \frac{h}{a} \left(= \frac{1}{\cos\theta}\right)\ }, \quad \csc(\theta) = \underline{\ \frac{h}{o} \left(= \frac{1}{\sin\theta}\right)\ }, \quad \cot(\theta) = \underline{\ \frac{a}{o} \left(= \frac{1}{\tan\theta}\right)\ }$$

In general, we allow arbitrary angle θ. Graphically, we have:



Some useful formulas:

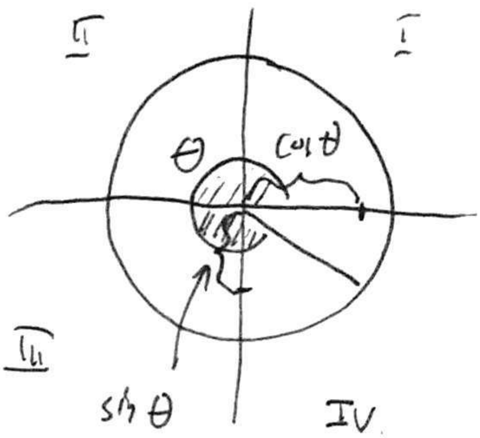
- $\cos^2\theta + \sin^2\theta = 1$
- $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$
- $\sec\theta = \frac{1}{\cos\theta}$, ... etc as in (*)

Exercise (standard values). Fill out the table below.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(\theta)$	0	$1/2$	$\frac{\sqrt{2}}{2}$	$\sqrt{3}/2$	1
$\cos(\theta)$	1	$\sqrt{3}/2$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	X

not defined!

Exercise: Given that θ is in the fourth quadrant and $\cos(\theta) = 4/5$, find the exact value of $\sec(\theta), \sin(\theta)$ and $\tan(\theta)$.



1) $\sec(\theta) = \frac{1}{\cos\theta} = \frac{1}{4/5} = \frac{5}{4}$

2) Kuchi:

1) $\sin^2\theta + \cos^2\theta = 1$

$\rightarrow \sin\theta = \pm\sqrt{1-\cos^2\theta}$

2) θ in 4th quadrant $\Rightarrow$ $\sin\theta < 0$

$\Rightarrow \sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(\frac{4}{5}\right)^2} = -\sqrt{\frac{25-16}{25}} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$

3) $\tan(\theta) = \frac{\sin\theta}{\cos\theta} = \frac{(-3/5)}{(4/5)} = -\frac{3}{4}$