MA 16010 Lesson 3: Limits Graphically

(if these exist)

Recall:

$\lim_{x\to c^-} f(x) =$ the value that $f(x)$ approaches as x approaches c on the left

$\lim_{x\to c^+} f(x) =$ the value that $f(x)$ approaches as x approaches c on the right

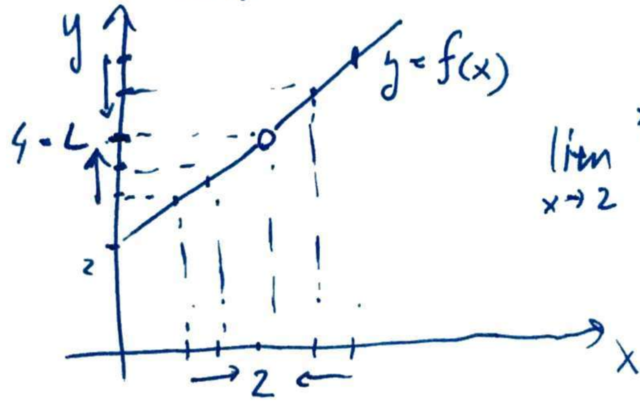
$\lim_{x\to c} f(x) = \lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$, given that these limits exist and agree.

How do limits (roughly) look like?

Example (finite limit - from last time). Consider

$\boxed{= x+2 \text{ for } x \neq 2}$

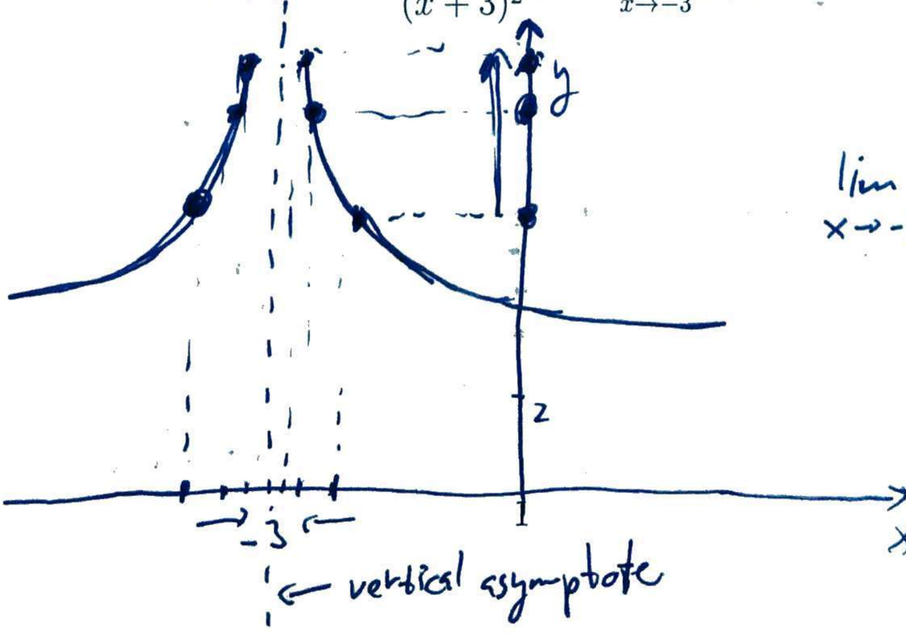
$$f(x) = \underbrace{\frac{x^2-4}{x-2}}, \quad \lim_{x\to 2} f(x) = ?$$



$$\lim_{x\to 2} \frac{x^2-4}{x-2} = 4$$

Example (infinite limit - from last time). Consider

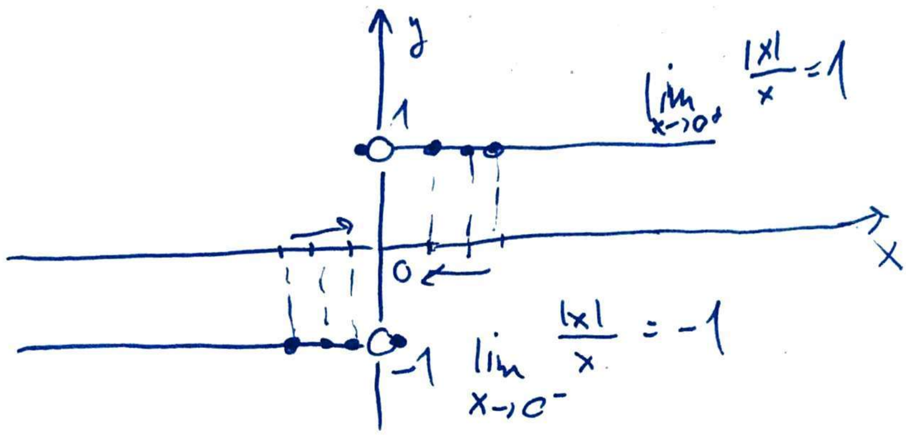
$$f(x) = 2 + \frac{4}{(x+3)^2}, \quad \lim_{x\to -3} f(x) = ?$$



$$\lim_{x\to -3}\left(2+\frac{4}{(x+3)^2}\right) = \infty$$

Example (one–sided limits - from last time). Consider

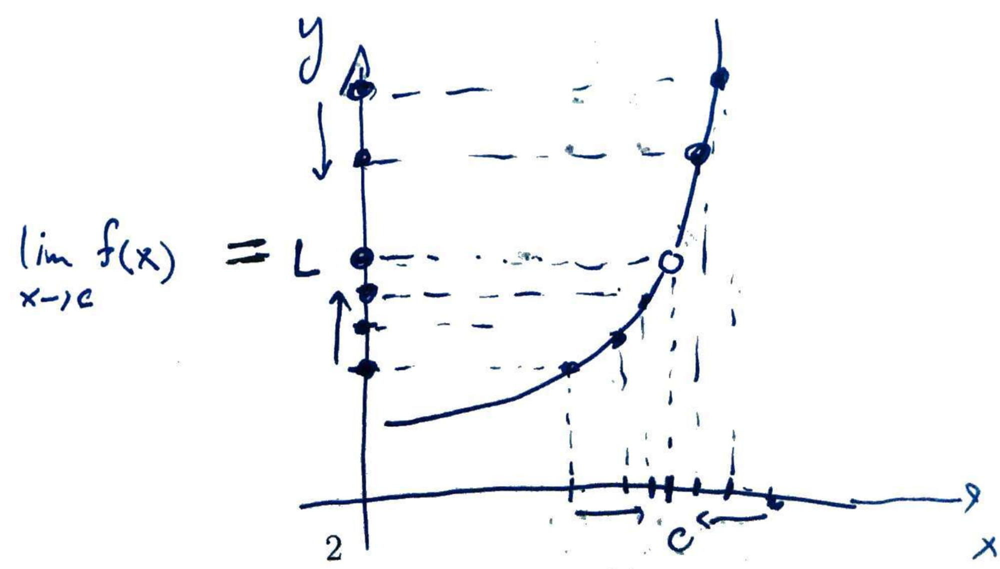
$$f(x) = \frac{|x|}{x}, \quad , \quad \lim_{x\to 0^-} f(x), \lim_{x\to 0^+} f(x) = ?$$



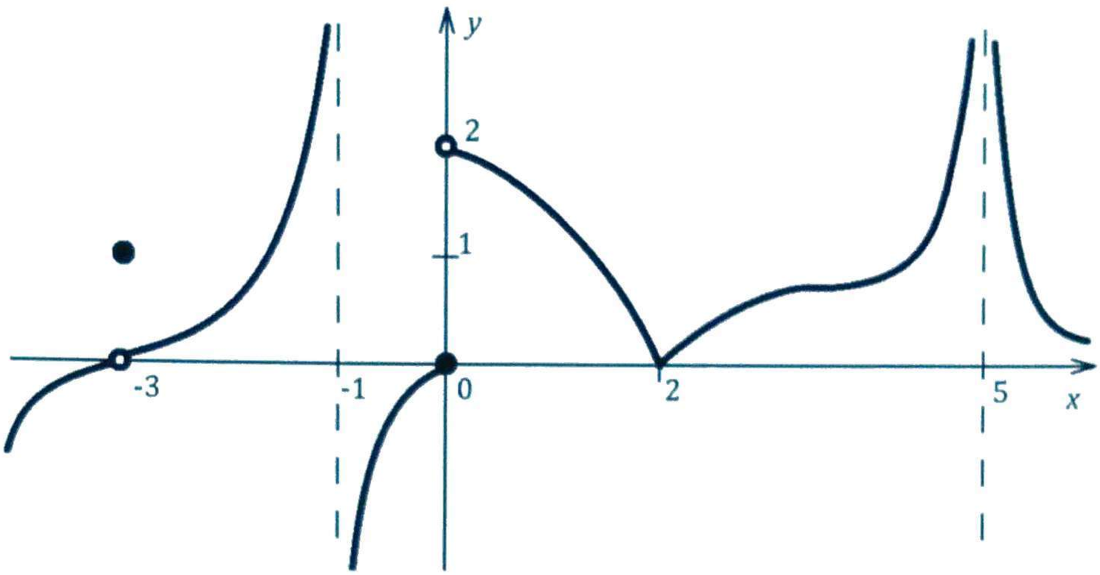
How to tell limits from the graph.

We want to find $\lim_{x\to c} f(x)$ based on the graph $y = f(x)$.

1. Locate c at the x–axis.
2. Look at x that approach c on the left or right, and locate their corresponding y–values.
3. Assuming it exists, $\lim_{x\to c} f(x)$ is the y–value around which the y–values from step 2. accumulate.



Exercise: Based on the sketch of the graph $y = f(x)$ below, find $\lim_{x\to c^-} f(x)$, $\lim_{x\to c^+} f(x)$, $\lim_{x\to c} f(x)$ and $f(c)$ for all c from the following list: -3, -1, 0, 2, 5. (In case some of the items do not exist, indicate that too.)



1) $\underline{c=-3}$

$$\left.\begin{array}{l}\lim_{x\to -3^-} f(x)=0 \\ \lim_{x\to -3^+} f(x)=0\end{array}\right\} \lim_{x\to -3} f(x)=0$$

$f(-3)=1$ ($\neq 0$!)

2) $\underline{c=-1}$

$$\left.\begin{array}{l}\lim_{x\to -1^-} f(x)=+\infty \\ \lim_{x\to -1^+} f(x)=-\infty\end{array}\right\} \lim_{x\to -1} f(x) \text{ DNE}$$

$f(-1)$ DNE (undefined)

3) $\underline{c=0}$

$$\left.\begin{array}{l}\lim_{x\to 0^-} f(x)=0 \\ \lim_{x\to 0^+} f(x)=2\end{array}\right\} \lim_{x\to 0} f(x) \text{ DNE}$$

$f(0)=0$ $(=\lim_{x\to 0_-} f(x))$ („f is left continuous at $x=0$")

4) $\underline{c=2}$:

$$\left.\begin{array}{l}\lim_{x\to 2^-} f(x)=0 \\ \lim_{x\to 2^+} f(x)=0\end{array}\right\} \lim_{x\to 2} f(x)=0$$

$f(2)=0=\lim_{x\to 2} f(x)$ „f is continuous at $x=2$"

5) $\underline{c=5}$

$$\left.\begin{array}{l}\lim_{x\to 5^-} f(x)=\infty \\ \lim_{x\to 5^+} f(x)=\infty\end{array}\right\} \lim_{x\to 5} f(x)=\infty$$

$f(5)$ DNE (undefined)