MA 16010 Lesson 30: Definite Integrals I

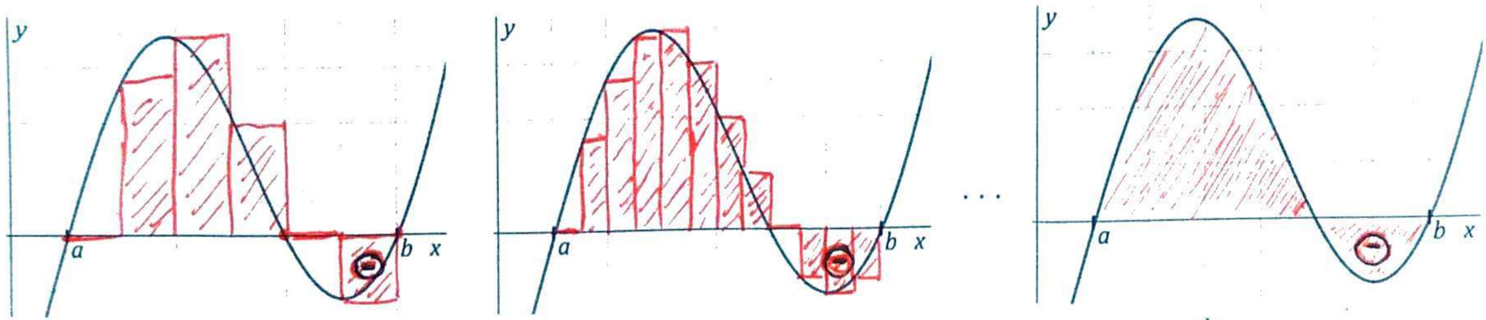
Recall: To approximate the signed area under the curve $y = f(x)$, over the interval $[a, b]$, we used **left/right Riemann sums**

$$L_n = \sum_{i=0}^{n-1} f(x_i)\cdot\Delta x \left(= \sum_{i=0}^{n-1} f(x_i)\cdot\frac{b-a}{n}\right) \quad R_n = \sum_{i=1}^{n} f(x_i)\Delta x \left(= \sum_{i=1}^{n} f(x_i)\frac{b-a}{n}\right)$$

As we increase n, the area is approximated better and better; to get the area precisely, we take the limit as $n \to \infty$.

We get $\int_a^b f(x)\,dx =$ "definite integral of $f(x)$ from a to b" $= \lim_{n\to\infty} \sum_{i=0}^{n-1} f(x_i)\Delta x =$

$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$ = the (signed) area under $y=f(x)$ over $[a,b]$.

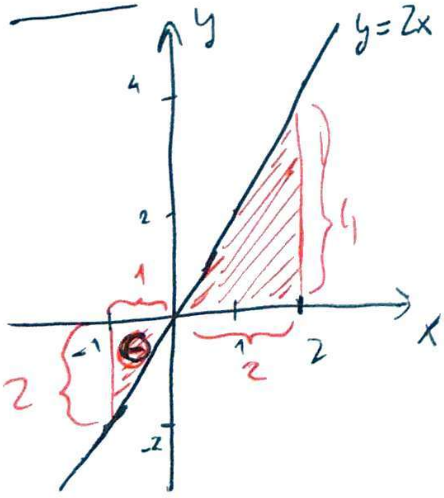


L_6 $\quad$ L_{12} $\quad$ $\cdots$ $\quad$ $n \to \infty$ $\quad$ $\int_a^b f(x)\,dx$

We can use geometric meaning of areas to "compute definite integrals".

Exercise: Evaluate $\displaystyle\int_{-1}^{2} 2x\,dx$ (by using geometric formulas).

Sketch:

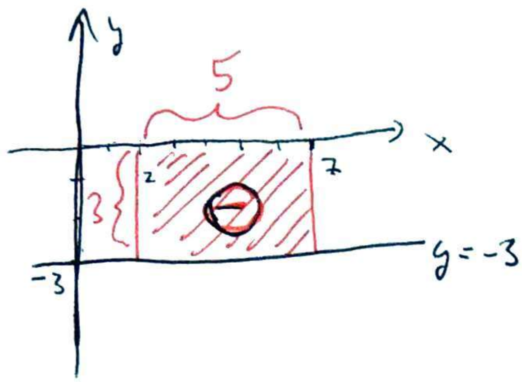


$\int_{-1}^{2} 2x\,dx =$ (area of the Δ on the right) $-$ (area of the Δ on the left)

$$= \frac{2\cdot 4}{2} - \frac{1\cdot 2}{2} = 4 - 1 = 3$$

Exercise: Evaluate $\displaystyle\int_2^7 -3\,\mathrm{d}x$ (by using geometric formulas).

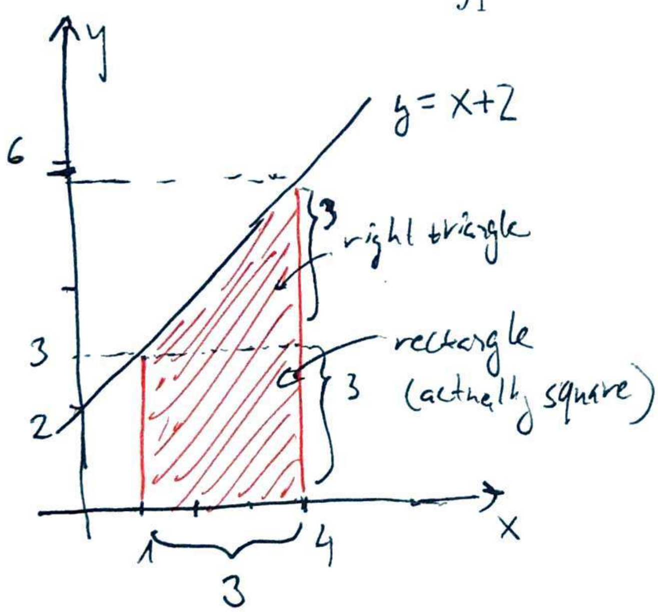
Sketch:



$$\int_2^7 -3\,dx = -(\text{area of the rectangle})$$

$$= -5 \cdot 3 = \underline{\underline{-15}}$$

Exercise: Evaluate $\displaystyle\int_1^4 (x+2)\,\mathrm{d}x$ (by using geometric formulas).



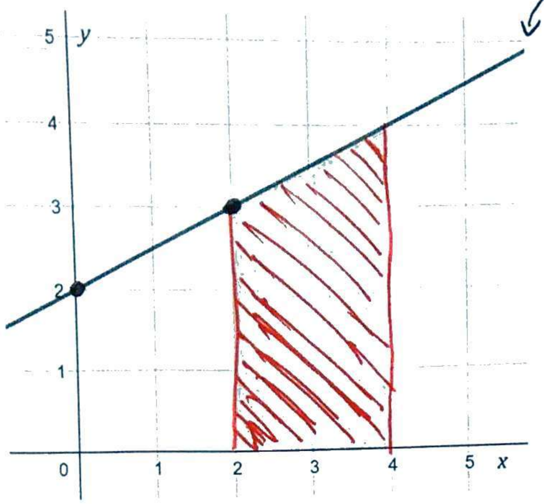
$$\int_1^4 (x+2)\,dx = (\text{area of the } \triangle) + (\text{area of the } \square)$$

$$= \frac{3 \cdot 3}{2} + 3 \cdot 3 =$$

$$= \frac{9}{2} + 9 = \frac{27}{2} = \underline{\underline{13.5}}$$

Exercise: Find the definite integral that expresses the (signed) area of the region sketched below.

need equation for this line



$y = ax + b$, we know e.g.: 1) $2 = a \cdot 0 + b$

→ $b = 2$

2) $3 = a \cdot 2 + 2$

$1 = 2a$

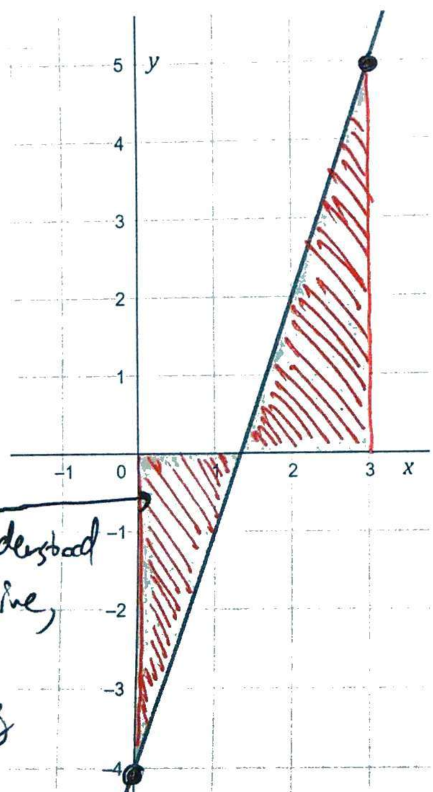
→ $a = \frac{1}{2}$

So $y = \frac{1}{2}x + 2$ is the line.

The area is expressed as $\int_2^4 \left(\frac{1}{2}x + 2\right)dx$

Exercise: Find the definite integral that expresses the (signed) area of the region sketched below.

Find the equation for the line:



this part is still understood as negative, despite it not being stated!

- put $y = ax + b$
- point $(0,-4)$: $-4 = a \cdot 0 + b$

$b = -4$

- point $(3,5)$: $5 = a \cdot 3 + (-4)$

$9 = a \cdot 3$

$a = 3$

→ $y = 3x - 4$,

and the area is expressed by the integral

$$\int_0^3 (3x - 4)\,dx$$