Quiz 11 Solution

1. Find the general solution to the equation

$$y''' - 3y'' + 2y' = 3e^{-t} \quad .$$

Solution: The auxiliary equation for

$$y''' - 3y'' + 2y' = 0$$

is

$$\lambda^3 - 3\lambda^2 + 2\lambda = 0$$
$$\lambda(\lambda - 1)(\lambda - 2) = 0$$

and so we have roots $\lambda_1 = 0, \lambda_2 = 1, \lambda_3 = 2,$

Thus, the general solution to the homogeneous equation is of the form

$$y_h = c_1 + c_2 e^t + c_3 e^{2t} \ .$$

To obtain a particular solution by the method of undetermined coefficients, we therefore seek a solution in the from

$$y_p = Ae^{-t}.$$

(Note that none of the roots matches the coefficient in the exponential e^{-t}, which is -1. Therefore the above form of particular solution should indeed work.)

To determine A we plug the expression into the original equation: we have $y_p' = -Ae^{-t}, y_p'' = Ae^{-t}, y_p''' = -Ae^{-t}$ and so we obtain

$$-Ae^{-t} - 3Ae^{-t} - 2Ae^{-t} = 3e^{-t}$$
$$-6Ae^{-t} = 3e^{-t}$$
$$-6A = 3 \ .$$

Therefore, $A = -1/2$, and the general solution to the equation is thus of the form

$$y = y_h + y_p = c_1 + c_2 e^t + c_3 e^{2t} - \frac{1}{2} e^{-t} \ .$$