Quiz 2 Solution

1. Find the general solution to the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y + xe^x \quad .$$

Solution: The equation is a linear differential equation. We rewrite it to its standard form,

$$\frac{\mathrm{d}y}{\mathrm{d}x} - y = xe^x,$$

and solve by the method of integrating factors. The integrating factor is

$$\mu(x) = e^{\int(-1)\mathrm{d}x} = e^{-x},$$

and so the solution is given by

$$y = \frac{\int \mu(x)xe^x \mathrm{d}x}{\mu(x)} = \frac{\int e^x(xe^x)\mathrm{d}x}{e^{-x}} = e^x\left(\int x\mathrm{d}x\right) = e^x\left(\frac{x^2}{2} + C\right) = \frac{x^2e^x}{2} + Ce^x,$$

where C is an arbitrary constant.

2. Determine whether the equation

$$(3x^2y - e^{xy})\mathrm{d}x + (x^3 - e^{xy})\mathrm{d}y = 0$$

is exact, and if it is, solve it.

Solution: We apply the test for exactness, i.e. compute $\frac{\partial}{\partial y}(3x^2y - e^{xy})$, $\frac{\partial}{\partial x}(x^3 - e^{xy})$ and compare them. We have

$$\frac{\partial}{\partial y}(3x^2y - e^{xy}) = 3x^2 - xe^{xy},$$

$$\frac{\partial}{\partial x}(x^3 - e^{xy}) = 3x^2 - ye^{xy}.$$

These two are not equal, so the equation is not exact.