Quiz 5 Solution

1. Decide whether the vector $\mathbf{b}$ is in the range of the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$, where

$$\mathbf{b} = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & -3 \\ 1 & 1 \\ 2 & -2 \end{bmatrix}.$$

Solution: The question is whether there is a vector $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{b}$. That is, we want to decide whether the matrix equation

$$\begin{bmatrix} 1 & -3 \\ 1 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}$$

has a solution $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. By row reduction of the augmented matrix for the system, we obtain:

$$\left[\begin{array}{cc|c} 1 & -3 & 2 \\ 1 & 1 & 2 \\ 2 & -2 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -3 & 2 \\ 0 & 4 & 0 \\ 2 & -2 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -3 & 2 \\ 0 & 4 & 0 \\ 0 & 4 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -3 & 2 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -3 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right].$$

We see from the last row that the given matrix equation does not have any solution. Thus, the conclusion is that $\mathbf{b}$ is not in range of the transformation T.

2. Compute the product:

$$\begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot (-3) + 2 \cdot 2 & 1 \cdot 4 + 2 \cdot 1 \\ (-1) \cdot (-3) + 1 \cdot 2 & (-1) \cdot 4 + 1 \cdot 1 \\ 2 \cdot (-3) + 0 \cdot 2 & 2 \cdot 4 + 0 \cdot 1 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 5 & -3 \\ -6 & 8 \end{bmatrix}.$$