Quiz 6 Solution

1. Compute the determinant

$$\begin{vmatrix} 0 & 3 & -1 \\ 5 & 2 & 4 \\ 1 & 4 & -2 \end{vmatrix}.$$

Solution: We know several methods of computation, and probably most people have seen the direct computation of 3×3 determinants before. So let us do it by row operations instead. First, switching two consecutive rows changes sign of the determinant, so we get

$$\begin{vmatrix} 0 & 3 & -1 \\ 5 & 2 & 4 \\ 1 & 4 & -2 \end{vmatrix} = - \begin{vmatrix} 0 & 3 & -1 \\ 1 & 4 & -2 \\ 5 & 2 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 4 & -2 \\ 0 & 3 & -1 \\ 5 & 2 & 4 \end{vmatrix}.$$

Next, subtracting a multiple of a row from another does not change the determinant. So we have

$$\begin{vmatrix} 1 & 4 & -2 \\ 0 & 3 & -1 \\ 5 & 2 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 4 & -2 \\ 0 & 3 & -1 \\ 0 & -18 & 14 \end{vmatrix} = \begin{vmatrix} 1 & 4 & -2 \\ 0 & 3 & -1 \\ 0 & 0 & 8 \end{vmatrix}.$$

Finally, a determinant of a matrix in its upper-triangular form is the product of its diagonal entries (this is a good rule to remember). So the determinant in the end is $1 \cdot 3 \cdot 8 = 24$.

2. Is the set of vectors

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; 2x - 3y + 2z = 3 \right\}$$

a vector subspace of $\mathbb{R}^3$? (Justify your answer!)

Solution: W is not a vector subspace of $\mathbb{R}^3$. Any vector subspace of $\mathbb{R}^3$ has to contain the zero vector $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. But since

$$2 \cdot 0 - 3 \cdot 0 + 2 \cdot 0 = 0 \neq 3,$$

the vector $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is not in W.