Quiz 7 Solution

1.Find the dimension of the subspace of $\mathbb{R}^3$ spanned by the vectors

$$\begin{bmatrix}1\\2\\4\end{bmatrix}, \begin{bmatrix}-1\\3\\6\end{bmatrix}, \begin{bmatrix}3\\1\\2\end{bmatrix} .$$

Solution: "Dimension of the column space does not change with row operations," so to find the dimension, we may row reduce the matrix whose columns are the given vectors:

$$\begin{bmatrix}1 & -1 & 3\\2 & 3 & 1\\4 & 6 & 2\end{bmatrix} \sim \begin{bmatrix}1 & -1 & 3\\0 & 5 & -5\\0 & 10 & -10\end{bmatrix} \sim \begin{bmatrix}1 & -1 & 3\\0 & 5 & -5\\0 & 0 & 0\end{bmatrix} \sim \begin{bmatrix}1 & -1 & 3\\0 & 1 & -1\\0 & 0 & 0\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 2\\0 & 1 & -1\\0 & 0 & 0\end{bmatrix} .$$

We see that the dimension fo the column space of the resulting matrix is 2 (e.g. because it has 2 pivot columns), so the dimension of the original subspace is also 2.

2. Find a basis of the space of all solutions of the equation

$$3x - 2y + 6z = 0 .$$

Solution: We can rewrite the equation as

$$x - \frac{2}{3}y + 2z = 0.$$

From this, we see that x is (can be taken as) the one pivot variable and the remaining variables y, z are free. That is, choosing $y = r, z = s$ as parameters for the solutions, the set of solutions is of the form

$$\left\{ \begin{bmatrix}\frac{2}{3}r - 2s\\r\\s\end{bmatrix} \middle| \, r, s \in \mathbb{R} \right\} = \left\{ r\begin{bmatrix}\frac{2}{3}\\1\\0\end{bmatrix} + s\begin{bmatrix}-2\\0\\1\end{bmatrix} \middle| \, r, s \in \mathbb{R} \right\}.$$

From this, we see that a basis for the set of solutions is $\left\{ \begin{bmatrix}\frac{2}{3}\\1\\0\end{bmatrix}, \begin{bmatrix}-2\\0\\1\end{bmatrix} \right\}$.