Quiz 8 Solution

1. Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 5 \\ 5 & -2 & 9 \\ 2 & -2 & 4 \end{bmatrix}.$$

Solution: By row operations we get

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 5 \\ 5 & -2 & 9 \\ 2 & -2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & -6 & 2 \\ 0 & -12 & 4 \\ 0 & -6 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & -6 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

We see that the matrix has two pivot columns, hence the rank of the matrix is 2.

2. Find all eigenvalues of the matrix

$$\begin{bmatrix} -2 & 2 \\ -2 & 3 \end{bmatrix}.$$

Solution: We have to compute the characteristic polynomial ($\det(A - \lambda I)$ where A is our matrix) and find its roots. We have

$$\begin{vmatrix} -2-\lambda & 2 \\ -2 & 3-\lambda \end{vmatrix} = (-2-\lambda)(3-\lambda) - 2(-2) = \lambda^2 - \lambda - 2.$$

The quadratic equation

$$\lambda^2 - \lambda - 2 = 0$$

has the roots given by the standard formula,

$$\lambda_{1,2} = \frac{1 \pm \sqrt{1 - 4(-2)}}{2} = \frac{1 \pm 3}{2}.$$

Thus, the eigenvalues are $\lambda_1 = -1$ and $\lambda_2 = 2$.