Quiz 9 Solution

1.Find the general solution to the equation

$$y'' - 2y' - 3y = 0 \ .$$

Solution: The auxiliary equation is

$$\lambda^2 - 2\lambda - 3 = 0$$
$$(\lambda - 3)(\lambda + 1) = 0$$

and so we see that it has two distinct roots $\lambda_1 = 3, \lambda_2 = -1$.

Thus, the general solution to the equation is of the form

$$y = c_1 e^{3t} + c_2 e^{(-1)t} = c_1 e^{3t} + c_2 e^{-t} \ .$$

2. Use the method of undetemined coefficients to find a solution to the equation

$$y'' - 2y' + y = t \ .$$

Solution: The right-hand side is of the form $t = te^{0t}$. After checking that 0 (coefficient in the exponential) is not a root of the auxilliary equation ($\lambda^2 - 2\lambda + 1 = 0$), we conclude that we are looking for a solution of the form

$$y_p = (A + Bt)e^{0t} = A + Bt.$$

What remains is to determine the coefficients A, B. We have

$$y_p' = B,$$
$$y_p'' = 0$$

Pluggin into the equation, we obtain

$$0 - 2B + (A + Bt) = t$$
$$(A - 2B) + Bt = t$$

Comparing the constant terms and the coefficients by t, we deduce

$$(A - 2B) = 0$$
$$B = 1 \ ,$$

so we obtain the coefficients $B = 1$ and $A = 2$. Thus we obtained a solution

$$y_p(t) = 2 + t \ .$$