MA 16010 Lesson 12: Chain rule II & logarithm

Recall (chain rule): Given two functions $y = f(u)$ and $u = g(x)$, for their composition $y = f(g(x))$ we have

$$y'(x) = \qquad\qquad\qquad\qquad, \quad \text{or} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \qquad\qquad\qquad\qquad.$$

Today we compute some more complicated derivatives, using all our rules.

Exercise: Compute the derivative of $h(x) = (3x-4)^4(x^3-5)^2$.

Exercise: Compute the derivative of $h(x) = \frac{\sqrt{16-x^2}}{3x}$ at $x = 1$.

Exercise: Compute the derivative of $h(x) = e^{5x} \cot(7x)$.

Derivative of the natural logarithm. We can use the chain rule to figure out what is the derivate of $y = \ln(x)$:

$$e^{\ln(x)} =$$

Summary: $$\frac{\mathrm{d}}{\mathrm{d}x}\Big[\ln(x)\Big] = \qquad .$$

Exercise: Compute $y'(2)$ when $y = \ln(x^3 + 3x)$.

Exercise: Compute $y'(x)$ when $y = \cos(2x^2 + 5)\ln(33x)$.

Exercise: Compute $h'(x)$ when $h(x) = 5^x$.

Exercise: The position of a particle moving on a straight line is given by

$$s(t) = \frac{50t}{(30 + 2t^2)^2}$$

(in meters, where t is time in seconds). What is the velocity of the particle at $t = 4$ seconds? Round your answer to 3 decimal places.