MA 16010 Lesson 27: Antiderivatives I

Example: If $f(x) = 3x^2 + x$ is the derivative of a function $F(x)$, then

$F(x) =$

In general, $F(x) =$

Antiderivatives. An antiderivative of a function $f(x)$ is a function $F(x)$ such that:

An antiderivative is determined only up to an additive constant. When $F(x)$ is an antiderivative of $f(x)$, we write:

We also call $F(x)$ ______________________________, and the process of finding it ________________________________.

Rules for integration. We can reverse engineer rules for antiderivatives from those for derivatives:

- **Additivity, constant multiples:**

- (**Product and chain rule:** MA26200, "int. by parts," "substitution")
- **Constant rule:**

- **Power rule:**

- **Antiderivatives of other functions:**

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big[\sin(x)\Big] = \cos(x) \quad \rightsquigarrow \quad \int \cos(x)\ \mathrm{d}x =$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big[\cos(x)\Big] = -\sin(x) \quad \rightsquigarrow \quad \int \sin(x)\ \mathrm{d}x =$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big[\ln(x)\Big] = \frac{1}{x},\ x > 0 \quad \rightsquigarrow \quad \int \frac{1}{x}\ \mathrm{d}x =$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big[e^x\Big] = e^x \quad \rightsquigarrow \quad \int e^x\ \mathrm{d}x =$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big[\tan(x)\Big] = \sec^2(x) \quad \rightsquigarrow \quad \int \sec^2(x)\ \mathrm{d}x =$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big[\cot(x)\Big] = -\csc^2(x) \quad \rightsquigarrow \quad \int \csc^2(x)\ \mathrm{d}x =$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big[\sec(x)\Big] = \sec(x)\tan(x) \quad \rightsquigarrow \quad \int \sec(x)\tan(x)\ \mathrm{d}x =$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big[\csc(x)\Big] = -\csc(x)\cot(x) \quad \rightsquigarrow \quad \int \csc(x)\cot(x)\ \mathrm{d}x =$$

Exercise: Compute

(a) $\displaystyle\int \frac{4x^3 + \sqrt[5]{x^3}}{x}\ \mathrm{d}x =$

Exercise (cont.): Compute

(b) $\displaystyle\int \sec(x)\Big(3\cos(x) - 5\tan(x)\Big)\,\mathrm{d}x =$

(c) $\displaystyle\int (x-2)^2\,\mathrm{d}x =$

(d) $\displaystyle\int \frac{2 - 3xe^x + \pi x\sin(x)}{x}\,\mathrm{d}x =$