MA 16010 Lesson 2: Limits Numerically

Limits.

Example. The function

$$f(x) = \frac{x^2 - 4}{x - 2}$$

is not defined at $x = 2$: $f(2) =$

We still wish to understand how the function behaves at least *near* $x = 2$. Let us list some values of $f(x)$ near $x = 2$:

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$				—			

We observe that as x approaches 2, the value $f(x)$ approaches __________.

We say that __________ is the *limit of* $f(x) = \frac{x^2-4}{x-2}$ *as* x *approaches* 2, also written as:

In general: We say that L is the limit of $f(x)$ as x approaches (a given number) c if ____.

We write this fact as

$$\lim_{x \to c} f(x) = L \quad .$$

Infinite limits: We can have $L = \infty$ or $L = -\infty$ in the above.

- $\lim_{x \to c} f(x) = \infty$ means: __________________________________.
- $\lim_{x \to c} f(x) = -\infty$ means: ________________________________.

Exercise: List the indicated values, rounded to 4 decimal places, and determine the indicated limit:

$$f(x) = \frac{\sin(x)}{x}$$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$				—			

$\lim_{x \to 0} \frac{\sin(x)}{x} =$

Exercise: List the indicated values, rounded to 4 decimal places, and determine the indicated limit:

$$f(x) = 2 + \frac{4}{(x+3)^2}$$

x	-3.1	-3.01	-3.001	-3	-2.999	-2.99	-2.9
$f(x)$				—			

$\lim_{x \to -3} \left(2 + \frac{4}{(x+3)^2}\right) =$

One–sided limits.
Example. Consider the function

$$f(x) = \frac{x^2}{2x-2}$$

and its behaviour near $x = 1$.

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$				—			

Does $\lim_{x \to 1} f(x)$ exist?

What can be said:

- As x approaches 1 *from the left / from below*, $f(x)$ approaches __________.

  We say that the left–sided limit of $f(x)$ as x approaches 1 (from the left) is equal to __________. We also write:

- As x approaches 1 *from the right / from above*, $f(x)$ approaches __________.

  We say that the right–sided limit of $f(x)$ as x approaches 1 (from the left) is equal to __________. We also write:

In general: We say that L is the limit of $f(x)$ as x approaches (a given number) c from the left / from the right if $f(z)$ tends to L as x approaches c from the left / from the right. We write this also as

$$\lim_{x \to c^-} f(x) = L \quad \text{and} \quad \lim_{x \to c^+} f(x) = L, \quad \text{resp.}$$

Relation to "both–sided limits": $\lim_{x \to c} f(x)$ exists and equals L if and only if:

Exercise: List the indicated values, rounded to 4 decimal places, and determine the indicated limit:

$$f(x) = \frac{x+2}{x^2 - 2x - 8}$$

x	4	4.0001	4.001	4.01	4.1
$f(x)$	—				

$\lim_{x \to 4^+} \frac{x+2}{x^2-2x-8} =$

Exercise: List the indicated values, rounded to 4 decimal places, and determine the indicated limits:

$$f(x) = \frac{|x|}{x}$$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$				—			

$\lim_{x \to 0^-} \frac{|x|}{x} =$

$\lim_{x \to 0^+} \frac{|x|}{x} =$

$\lim_{x \to 0} \frac{|x|}{x} =$