MA 16010 Lesson 31: Definite Integrals II

Recall: The geometric meaning of the definite integral $\int_a^b f(x)\,\mathrm{d}x$ is:

"Algebraic rules" for definite integrals. Assume $a \leq b \leq c$.

1. $\displaystyle\int_a^a f(x)\,\mathrm{d}x =$

2. $\displaystyle\int_a^b f(x)\,\mathrm{d}x + \int_b^c f(x)\,\mathrm{d}x =$

3. $\displaystyle\int_b^a f(x)\,\mathrm{d}x =$ (*"convention"*)

4. $\displaystyle\int_a^b \big(f(x) \pm g(x)\big)\,\mathrm{d}x =$

5. $\displaystyle\int_a^b \big(k \cdot f(x)\big)\,\mathrm{d}x =$ (k a constant)

Exercise: Assuming that $\displaystyle\int_2^4 6x^2\,\mathrm{d}x = 112$,

(a) find $\displaystyle\int_4^2 6x^2\,\mathrm{d}x$:

(b) find $\displaystyle\int_4^2 15x^2\,\mathrm{d}x$:

Exercise: Given that $\int_0^3 x^2 \, \mathrm{d}x = 9$, $\int_3^6 x^2 \, \mathrm{d}x = 63$ and $\int_0^6 x^3 \, \mathrm{d}x = 324$,

find $\int_0^6 \left(4x^2 - x^3\right) \mathrm{d}x$.

Exercise: Given that $\int_{-4}^{7} f(t)\mathrm{d}t = 31$, $\int_{-4}^{-1} f(t)\mathrm{d}t = 8$ and $\int_{-1}^{7} g(t)\mathrm{d}t = 11$,

find $\int_{-1}^{7} \left(g(t) - 2f(t)\right) \mathrm{d}t$.

Exercise: Given that $\displaystyle\int_a^b f(x)\mathrm{d}x = 14$ and $\displaystyle\int_a^c f(x)\mathrm{d}x = 2\cdot\int_c^b f(x)\mathrm{d}x$,

find $\displaystyle\int_c^b f(x)\mathrm{d}x$.