MA 16010 Lesson 33: Fundamental Theorem of Calculus II

Recall (fundamental theorem of calculus): If $y = f(x)$ is a function that is continuous on $[a, b]$ then

$$\int_a^b f(x)\,\mathrm{d}x = \qquad ,$$

where $F(x)$ is

What are definite integrals good for:

They measure how certain qunatities "accumulate over time".

Example: A company maintains a website through a web hosting provider. The expected hosting cost per year, in dollars per year, is

$$c(t) = 500 + \frac{300}{\sqrt{t}} \ ,$$

where t is time in years from now. How much does the company expect to pay for hosting of their website over the next 10 years?

Exercise: A water pipe bursts in the bathroom at 9:05 am, and as a result, water floods the bathroom at the rate

$$r(t) = 70 + 3\sqrt{t} \text{ gal/min},$$

where t is the time in minutes since 9:05 am.

(a) How much water poured into the bathroom within the first hour, i.e. between 9:05 am and 10:05 am?

(b) How much water poured into the bathroom during the second hour, i.e. between 10:05 am and 11:05 am?

Exercise: The acceleration of a car t seconds after hitting the brakes is

$$a(t) = -5 - \frac{t}{5}$$

miles per hour per second.

(a) What is the decrease in velocity 5 seconds after after hitting the brakes?

(b) Given that the initial velocity of the car was 50 mph, how much time is needed before the car comes to a halt?