

MA 16010 Lesson 36: Exponential Decay

Recall: The solution to the equation $y' - ky$ is:

when $k > 0$, we speak of:

Today: We consider the case $k < 0$. Then we speak of:

typical situation:

Example: The amount $A(t)$ of a radioactive isotope (that decays over time) obeys the equation

$$\frac{dA}{dt} = -0.0002A$$

(where t is time in years). How long does it take for an initial amount $A(0)$ of the isotope to be reduced to half?

The time that we obtained in the previous problem as the (aptly named)

Exercise: The radioactive isotope ^{226}Ra has a half-life of approximately 1599 years. There are 210g of ^{226}Ra now. How much of ^{226}Ra is left after 15000 years?

Exercise: A drug in a patient's body has half-life of 7 hours. If a patient takes a dose of 500 mg at 9:00 am, how much of the drug remains in his system at 9:00 am the next day?

Carbon dating.

The isotope ^{14}C (Carbon-14) is created in the atmosphere due to cosmic rays. Plants incorporate it during photosynthesis, and as a result, living organisms naturally contain ^{14}C . Once the plant or animal dies, the concentration of ^{14}C starts decaying. The half-life of ^{14}C is 5,730 years.

Exercise: An ancient mammal bone contains 2 mg of ^{14}C . Based on the size of the bone, we estimate that the bone contained 250 g of ^{14}C when the mammal was alive. Approximately how long ago did the animal die?