MA 16010 Lesson 3: Limits Graphically

Recall:

$\lim_{x \to c^-} f(x) =$ ______________________________

$\lim_{x \to c^+} f(x) =$ ______________________________

$\lim_{x \to c} f(x) =$ ______________________________

How do limits (roughly) look like?

Example (finite limit - from last time). Consider

$$f(x) = \frac{x^2 - 4}{x - 2}, \quad \lim_{x \to 2} f(x) = ?$$

Example (infinite limit - from last time). Consider

$$f(x) = 2 + \frac{4}{(x+3)^2}, \quad , \quad \lim_{x \to -3} f(x) = ?$$

Example (one–sided limits - from last time). Consider

$$f(x) = \frac{|x|}{x}, \quad , \quad \lim_{x \to 0^-} f(x), \lim_{x \to 0^+} f(x) = ?$$

How to tell limits from the graph.

We want to find $\lim_{x \to c} f(x)$ based on the graph $y = f(x)$.

1. Locate c at the x–axis.
2. Look at x that approach c on the left or right, and locate their corresponding y–values.
3. Assuming it exists, $\lim_{x \to c} f(x)$ is the y–value around which the y–values from step 2. accumulate.

Exercise: Based on the sketch of the graph $y = f(x)$ below, find $\lim_{x\to c^-} f(x)$, $\lim_{x\to c^+} f(x)$, $\lim_{x\to c} f(x)$ and $f(c)$ for all c from the following list: $-3,\ -1,\ 0,\ 2,\ 5$. (In case some of the items do not exist, indicate that too.)