MA 16010 Lesson 9: Product rule

Recal: Computational rules for limits that we know so far are:

1. **Constant rule:** $\frac{\mathrm{d}}{\mathrm{d}x}\left[c\right] =$

2. **Power rule:** $\frac{\mathrm{d}}{\mathrm{d}x}\left[x^n\right] =$

3. **Sum, difference rules:** $\frac{\mathrm{d}}{\mathrm{d}x}\left[f(x) \pm g(x)\right] =$

4. **Derivatives of basic functions:**

$\frac{\mathrm{d}}{\mathrm{d}x}\left[\sin(x)\right] =$ $\qquad$ $\frac{\mathrm{d}}{\mathrm{d}x}\left[\cos(x)\right] =$ $\qquad$ $\frac{\mathrm{d}}{\mathrm{d}x}\left[e^x\right] =$

Today we add a new one:
5. **Product rule:** Given two functions $f(x), g(x)$, we have

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[f(x) \cdot g(x)\right] =$$

Exercise: Compute $f'(x)$ when $f(x) = (3x^2 - 4)(x + 2\sqrt{x})$.

(a) With the product rule:

(b) Without the product rule:

Exercise: Compute $y'(\pi)$ when

$$y = \frac{\sin(x)}{x^2} \quad :$$

Exercise: Find all x where the graph of the function $f(x)$ has a horizontal tangent line, where

$$f(x) = (x^2 - 2x)e^x \quad .$$

Exercise: Find the equation of the tangent line to $y = 3x\sin(x)$ at $x = \pi/2$.

Exercise: Compute $f'(x)$ when

(a) $f(x) = e^{2x}$:

(b) $f(x) = \sin(2x)$:

EXTRAS (<u>not needed</u> for homework or class; purely for anyone's interest)

1. Justify the product rule by continuing the limit computation (you may assume that $f(x), g(x)$ are continuous):

$$\frac{\mathrm{d}}{\mathrm{d}x}\big[f(x)\cdot g(x)\big] = \lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h} =$$
$$= \lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x+h)+f(x)g(x+h)-f(x)g(x)}{h} =$$
$$= \cdots$$

2. Deduce the quotient rule: using that $g(x)\cdot\frac{f(x)}{g(x)} = f(x)$ and the product rule, deduce the formula for $\frac{\mathrm{d}}{\mathrm{d}x}\big[\frac{f(x)}{g(x)}\big]$.