MA 16020 Lesson 11: Areas between curves

Recall: The geometric meaning of the integral $\int_a^b f(x)\mathrm{d}x$ is:

Suppose that we have two functions $f(x), g(x)$ such that $f(x) > g(x)$ on a given interval $[a, b]$.
Question 1: How to compute the area between the graphs of $f(x)$ and $g(x)$ over the interval $[a, b]$?

Question 2: What if the functions $f(x), g(x)$ "cross each other"?

Exercise 1. Sketch the region and set up an integral computing the area
(a) between the curves $y = 3x$, $y = 2x$ over the interval $2 \leq x \leq 4$:

(b) enclosed by the curves $y = x^4$, $y = 4x$:

(c) between the curves $y = \cos(x)$, $y = 1/2$ over the interval $0 \leq x \leq \pi/2$:

Exercise 2. A company installs in its factory new machines that are expected to provide income at the rate of $105000 - 100t^2$ dollars per year, where t is the number of years since installation. On the other hand, the maintenance cost for the machines is $500t$ dollars per year. What is the overall profit of the company from these machines before they need to be replaced?

Exercise 3. A company expects growth of its profits at the continuous rate between 5% and 15%. The company's profit within the past year was 21 million dollars. Find the positive cumulative difference in predicted total profits over the next 3 years.

Exercise 4 (if time permits). Find the equation of the vertical line that divides the area of the region enclosed by the curves $y = 6x - x^2$, $y = x^2 - 4x$ in half.