

MA 16020 Lesson 15: Improper integrals

Recall (limits): The limit of the function $f(x)$ as x approaches a , $\lim_{x \rightarrow a} f(x)$, is a value L such that:

An *improper integral* is a definite integral $\int_a^b f(x)dx$ such that the integrand $f(x)$ is defined on (a, b) , but not necessarily at a or b .

Example. Evaluate the integral

$$\int_1^{\infty} \frac{dx}{x^3}.$$

Key idea: The integral $\int_a^b f(x)dx$ can be computed as

and/or

Exercise 1. Evaluate the integral

$$\int_8^{\infty} \frac{5dx}{x(\ln(x))^3}.$$

Exercise 2. Evaluate the integral

$$\int_4^{\infty} \frac{dx}{\sqrt{x-3}}.$$

Exercise 3. Evaluate the integral

$$\int_5^{11} \frac{3dx}{\sqrt[3]{x-5}} .$$

Exercise 4. Evaluate the integral

$$\int_4^{\infty} \frac{3e^{-\sqrt{x}}}{2\sqrt{x}} dx .$$

Exercise 5. Evaluate the integral

$$\int_0^{2\pi} \tan\left(\frac{\theta}{4}\right) d\theta .$$

Exercise 6. Evaluate the integral

$$\int_2^{\infty} \frac{dx}{x \ln(2x^3)} .$$