MA 16020 Lesson 16: Geometric series I

Series. A (number) series is:

Example:

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$$
$$\sum_{n=1}^{\infty} \frac{3}{n^2}$$
$$\sum_{n=0}^{\infty} (n+2)$$

A **partial sum** of a series is:

Example: For the series

$$\sum_{n=0}^{\infty} \frac{3}{n^2+n+2},$$

the first partial sum is $S_0 =$

the second partial sum is $S_1 =$

the third partial sum is $S_2 =$

The sum of a series is:

A series is called **convergent** if

A series is called **divergent** if

A **geometric series** is:

Finding the **sum of a geometric series:**

The above works when the geometric series is **convergent**, which happens if and only if __________________________.

If __________________________, the geometric series is **divergent**.

Example. Decide whether the series

$$\sum_{n=0}^{\infty} \frac{5 \cdot 4^{n+1}}{7^n}$$

is geometric and convergent, and if it is, find its sum.

Exercise 1. Write

$$4 - \frac{8}{3} + \frac{16}{9} - \frac{32}{27} + \dots$$

in a compact form.

Exercise 2. Write the number

$$17.\overline{17} = 17.1717171717\dots$$

in the form of a series.

Exercise 3. Compute
$$\sum_{n=0}^{\infty} \left(\frac{3}{4^n} - \frac{6}{5^n} \right).$$

Exercise 4. Approximate the sum
$$\sum_{n=0}^{\infty} \frac{e^{-2n}}{4}$$
to 4 decimal places.